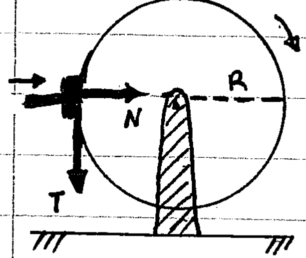
3.1.

$M = 2\,kg$
$R = 0{,}5\,m$
$\mu = 0{,}25$

A. Πληροφορίες από γραφική παράσταση $\omega = f(t)$:

- $\omega_0 = 50\ rad/s$
- $t_{ολ} = 10 s$, $t_{ολ} = \frac{\omega_0}{|\alpha_{γων}|} \Rightarrow |\alpha_{γων}| = 5\ rad/s^2$
- Εμβαδό $= \theta_{ολ} \Rightarrow \theta_{ολ} = \frac{50 \cdot 10}{2}$ ή $\theta_{ολ} = 250\,rad$



- $\Sigma\tau = I \cdot \alpha_{γων} \Rightarrow T \cdot R = \frac{1}{2} M R^2 \cdot \alpha_{γων} \Rightarrow T = 2{,}5 N$
- $T = \mu \cdot N \Rightarrow \boxed{N = 10 N}$

B. • $L = I\omega \Rightarrow L = \frac{1}{2} M R^2 \omega \Rightarrow \omega = \frac{2L}{MR^2} \Rightarrow \omega = 20\ rad/s$

- $\frac{\alpha_ε}{\alpha_κ} = \frac{\alpha_{γων} R}{\frac{\upsilon^2}{R}} \overset{\upsilon = \omega R}{=\!=\!=} \frac{\alpha_{γων} \cdot R}{\omega^2 \cdot R}$ ή $\boxed{\frac{\alpha_ε}{\alpha_κ} = \frac{1}{80} = 125 \cdot 10^{-4}}$

Γ. • $\omega = \omega_0 - |\alpha_{γων}| t_1 \Rightarrow t_1 = \frac{\omega_0 - \omega}{|\alpha_{γων}|} \Rightarrow t_1 = 6s$

- $\theta = \omega_0 t_1 - \frac{1}{2} |\alpha_{γων}| \cdot t_1^2 \Rightarrow \theta = 210\ rad$
- $W = -\tau_T \cdot \theta = -T \cdot R \cdot \theta = -2{,}5 \cdot 0{,}5 \cdot 210\, J$ ή $\boxed{W = -262{,}5\,J}$

■ 2ος Τρόπος: $\Sigma W = \Delta K \Rightarrow W = K_{τελ} - K_{αρχ} \Rightarrow W = \frac{1}{2} I \omega^2 - \frac{1}{2} I \omega_0^2 \Rightarrow$
$\Rightarrow W = -262{,}5\,J.$

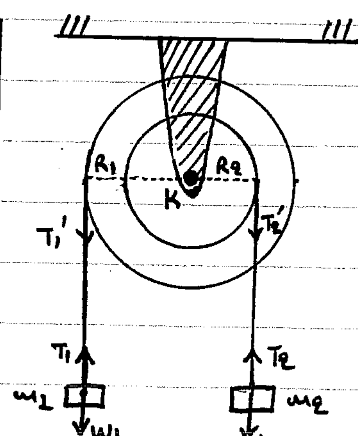
- $P = \tau_T \cdot \omega = -T \cdot R \cdot \omega = -2{,}5 \cdot 0{,}5 \cdot 20\, W$ ή $\boxed{P = -25 W}$

Δ. • Από το εμβαδό της γραφικής παράστασης $\omega = f(t)$ μπορούμε να υπολογίσουμε ότι $\boxed{\theta_{ολ} = 250\ rad}$

- $\bar{P} = \frac{W_{ολ}}{t_{ολ}} = \frac{-T \cdot R \cdot \theta_{ολ}}{t_{ολ}}$ ή $\boxed{\bar{P} = -31{,}25\,W}$

3.2

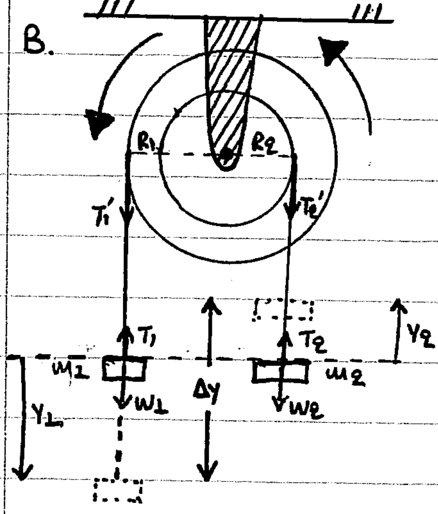
$R_1 = 0{,}4\,m$
$R_2 = 0{,}2\,m$
$m_1 = 2\,kg$
m_2
$I_{cm} = \frac{1}{2} M R^2$
$g = 10\,m/s^2$



A. Το σύστημα ισορροπεί:

- Σώμα m_1: $\Sigma F = 0 \Rightarrow T_1 - w_1 = 0 \Rightarrow$
  $\Rightarrow T_1 = m_1 g = 20 N \ (= T_1')$
- Τροχαλία: $\Sigma\tau = 0 \Rightarrow T_1' R_1 - T_2' R_2 = 0$
  $\Rightarrow T_2' = T_1' \frac{R_1}{R_2} \Longrightarrow$
  $\Rightarrow T_2' = 40 N \ (= T_2)$
- Σώμα m_2: $\Sigma F = 0 \Rightarrow T_2 - w_2 = 0$
  $\Rightarrow T_2 = m_2 g \Rightarrow \boxed{m_2 = 4\,kg}$

Β.



- $\theta = N \cdot 2\pi \Rightarrow \theta = \frac{50}{\pi} \cdot 2\pi\, rad \Rightarrow \underline{\theta = 100\, rad}$
- $\theta = \frac{1}{2} \alpha_{γων} t^2 \Rightarrow \underline{\alpha_{γων} = 2\, rad/s^2}$
- $\alpha_{cm(1)} = \alpha_{γων} \cdot R_1 \Rightarrow \underline{\alpha_{cm(1)} = 0{,}8\, m/s^2}$

  $\alpha_{cm(2)} = \alpha_{γων} R_2 \Rightarrow \underline{\alpha_{cm(2)} = 0{,}4\, m/s^2}$
- Σώμα m_1: $\Sigma F = m_1 \cdot \alpha_{cm(1)} \Rightarrow W_1 - T_1 = m_1 \cdot \alpha_{cm(1)}$

  $\Rightarrow T_1 = m_1 g - m_1 \cdot \alpha_{cm(1)} \Rightarrow \underline{T_1 = 18{,}4 N}$
- Σώμα m_2: $\Sigma F = m_2 \cdot \alpha_{cm(2)} \Rightarrow T_2 - W_2 = m_2 \cdot \alpha_{cm(2)}$

  $\Rightarrow T_2 = m_2 \cdot g + m_2 \cdot \alpha_{cm(2)} \Rightarrow \underline{T_2 = 15{,}6 N}$
- Τροχαλία: $\Sigma \tau = I \cdot \alpha_{γων} \Rightarrow T_1' \cdot R_1 - T_2' R_2 = I \cdot \alpha_{γων}$

$$\xrightarrow{T_1 = T_1' \text{ και } T_2 = T_2'} I = \frac{18{,}4 \cdot 0{,}4 - 15{,}6 \cdot 0{,}2}{2}\, kg \cdot m^2 \Rightarrow \boxed{I = 2{,}12\, kg \cdot m^2}$$

Γ. $I = \frac{1}{2} M R_1^2 + \frac{1}{2} M R_2^2 \Rightarrow I = \frac{1}{2} M (R_1^2 + R_2^2) \Rightarrow M = \frac{2 \cdot I}{R_1^2 + R_2^2} \Rightarrow \boxed{M = 21{,}2\, kg}$

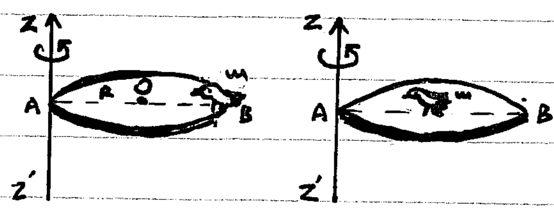
Δ. • $y_1 = \frac{1}{2} \alpha_{cm(1)} \cdot t^2 \xrightarrow{t=1s} y_1 = 0{,}4 m$

$y_2 = \frac{1}{2} \alpha_{cm(2)} \cdot t^2 \xrightarrow{t=1s} y_2 = 0{,}2 m$ $\Big\} \Rightarrow \Delta y = y_1 + y_2 \Rightarrow \boxed{\Delta y = 0{,}6 m}$

- $Π\% = \frac{K_{τρ}}{W_{w_1}} \cdot 100\% \Rightarrow Π\% = \frac{\frac{1}{2} I \omega^2}{w_1 \cdot y_1} \cdot 100\% \xrightarrow{\omega = \alpha_{γων} t = 2\, rad/s} Π\% = \frac{\frac{1}{2} \cdot 2{,}12 \cdot 4}{20 \cdot 0{,}4} \cdot 100\%$

  $\Rightarrow \boxed{Π\% = 53\%}$.

3.3 $\boxed{M = 0{,}4 kg,\ R = 0{,}1 m \\ m = 0{,}1 kg,\ \omega_1 = 7 \frac{rad}{s}}$ A.



Αφού $\Sigma \tau_{εξ} = 0$ ισχύει η Αρχή Διατήρησης της Στροφορμής

- Α.Δ.Στρ: $\vec{L}_{αρχ} = \vec{L}_{τελ}$

  $I_1 \omega_1 = I_2 \omega_2 \Rightarrow \omega_2 = \frac{I_1}{I_2} \cdot \omega_1$ (1)
- $I_1 = I_B^{δίσκου} + I_B^{m}$

  Θ. Steiner: $I_B^{δίσκου} = I_{cm} + MR^2 = \frac{3}{2} MR^2$ $\Big\} \Rightarrow I_1 = \frac{3}{2} MR^2 + m 4R^2 \Rightarrow \underline{I_1 = 0{,}01\, kgm^2}$
- $I_2 = I_B^{δίσκου} + I_B^{m} \Rightarrow I_2 = \frac{3}{2} MR^2 + mR^2 \Rightarrow \underline{I_2 = 0{,}007\, kgm^2}$
- (1) $\Rightarrow \omega_2 = \frac{0{,}01}{0{,}007} \cdot 7\ rad/s \Rightarrow \boxed{\omega_2 = 10\, rad/s}$

Β. $\Delta K = K_{τελ} - K_{αρχ} \Rightarrow \Delta K = \frac{1}{2} I_2 \omega_2^2 - \frac{1}{2} I_1 \omega_1^2 \Rightarrow \Delta K = \left(\frac{1}{2} 7 \cdot 10^{-3} \cdot 10^2 - \frac{1}{2} 10^{-2} \cdot 49\right) J \Rightarrow$

$\Rightarrow \Delta K = (0{,}35 - 0{,}245)\, J \Rightarrow \boxed{\Delta K = 0{,}105\, J}$

Γ. · $\Sigma\tau = I_2 \cdot a_{\gamma\omega\nu} \Rightarrow -F \cdot 2R = I_2 \cdot a_{\gamma\omega\nu} \Rightarrow a_{\gamma\omega\nu} = -\frac{7\cdot10^{-2}\cdot 2\cdot 10^{-1}}{7\cdot 10^{-3}}\ rad/s^2$

$\Rightarrow \boxed{a_{\gamma\omega\nu} = -2\ rad/s^2}$

· $\Delta\theta = \frac{\omega_2^2}{2|a_{\gamma\omega\nu}|} \Rightarrow \Delta\theta = \frac{10^2}{4}\ rad \Rightarrow \boxed{\Delta\theta = 25\ rad}$

· $t = \frac{\omega_2}{|a_{\gamma\omega\nu}|} \Rightarrow t = \frac{10}{2}\ s \Rightarrow \boxed{t = 5s}$

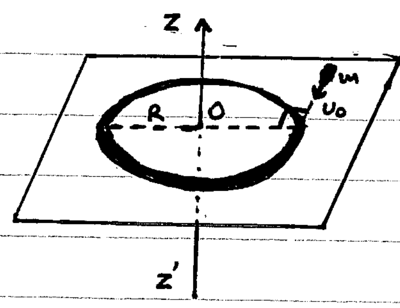
Δ. Ο ρυθμός μεταβολής της κινητικής ενέργειας του συστήματος είναι:

$$\frac{dK}{dt} = \Sigma\tau \cdot \omega \xrightarrow{\omega = \omega_2 - a_{\gamma\omega\nu} t_1 = 5\ rad/s} \frac{dK}{dt} = -F \cdot 2R \cdot \omega$$

$$\Rightarrow \frac{dK}{dt} = -0{,}07 \cdot 0{,}2 \cdot 5\ J/s \quad \text{ή} \quad \boxed{\frac{dK}{dt} = -0{,}07\ J/s}$$

3.4. $M = 3{,}6kg$
$R = 0{,}2m$
$m = 0{,}2kg$
υ_0
$\omega = 20\ rad/s$

Α.



$I_0 = I_{cm}^{\delta\acute{\iota}\sigma\kappa o\upsilon} + I_0^{m} \Rightarrow$

$I_0 = \frac{1}{2}MR^2 + mR^2 \Rightarrow$

$I_0 = (\frac{1}{2}3{,}6 \cdot 0{,}04 + 0{,}2 \cdot 0{,}04)\ kgm^2 \Rightarrow$

$\boxed{I_0 = 0{,}08\ kgm^2}$

Β. Αφού για το σύστημα δίσκος - βλήμα ισχύει $\Sigma\tau_{\epsilon\xi} = 0$, μπορούμε να εφαρμόσουμε την αρχή διατήρησης της στροφορμής:

$\vec{L}_{\alpha\rho\chi} = \vec{L}_{\tau\epsilon\lambda}$

$m\upsilon_0 R = I_0 \cdot \omega \Rightarrow \upsilon_0 = \frac{I_0 \cdot \omega}{mR} \Rightarrow \upsilon_0 = \frac{0{,}08 \cdot 20}{0{,}2 \cdot 0{,}2}\ m/s \Rightarrow \boxed{\upsilon_0 = 40\frac{m}{s}}$

Γ. · $\Sigma\tau = I_0 \cdot a_{\gamma\omega\nu} \Rightarrow -F \cdot R = I_0 \cdot a_{\gamma\omega\nu} \Rightarrow a_{\gamma\omega\nu} = -\frac{8 \cdot 0{,}2}{0{,}08}\ \frac{rad}{s^2} \Rightarrow$

$\Rightarrow a_{\gamma\omega\nu} = -20\ rad/s^2$

· $t_{o\lambda} = \frac{\omega_0}{|a_{\gamma\omega\nu}|} \xrightarrow{\omega_0 = 20\ rad/s} \boxed{t_{o\lambda} = 1s}$

Δ. · $\omega_1 = \omega_0 - |a_{\gamma\omega\nu}| t_1 \xrightarrow{\omega_0 = 20 rad/s} \omega_1 = (20 - 20 \cdot 0{,}5)\ rad/s \Rightarrow \omega_1 = 10\ rad/s$

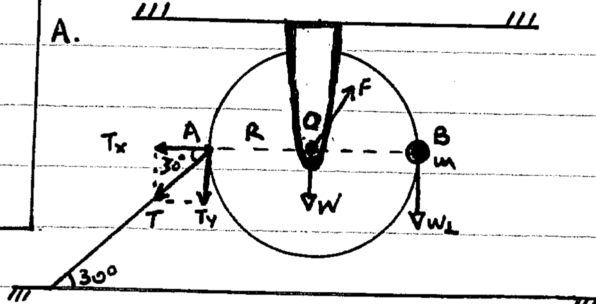
· $K = \frac{1}{2} I_0 \omega_1^2 \Rightarrow K = (\frac{1}{2} 0{,}08 \cdot 10^2)\ J \Rightarrow \boxed{K = 4J}$

· $\frac{K_{\alpha\rho\chi} - K_{\tau\epsilon\lambda}}{\Delta t} = -\Sigma\tau \cdot \omega_1 = -(-F \cdot R) \cdot \omega_1 = 8 \cdot 0{,}2 \cdot 10\ J/s \Rightarrow$

$\Rightarrow \boxed{\frac{K_{\alpha\rho\chi} - K_{\tau\epsilon\lambda}}{\Delta t} = 16\ J/s}$

3.5 $M = 18\,kg$, $R = 1\,m$, $T_{θρ} = 20\,N$, m

A.



Το σύστημα τροχαλία - μάζα m ισορροπεί:

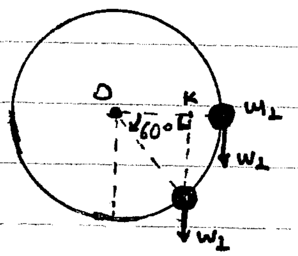
$\Sigma\tau_{(O)} = 0 \Rightarrow T_y \cdot R - W_1 \cdot R = 0$

$\Rightarrow W_1 R = T_y \cdot R \xrightarrow{T_y = T \cdot ημθ}$

$\Rightarrow mg = T \cdot ημ30 \Rightarrow T = 2mg$ ①

Το σκοινί σπάει για $T \geq T_{θρ} \xrightarrow{①} 2mg \geq T_{θρ} \Rightarrow m \geq \frac{T_{θρ}}{2g} \Rightarrow \boxed{m_1 = 1\,kg}$

B. 1)



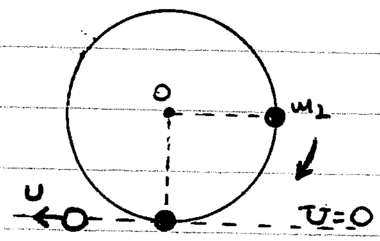
$\Sigma\tau_{(O)} = I_O \cdot a_{γων}$ ②

$I_O = I_{cm}^{τροχαλίας} + m_1 R^2 = \frac{1}{2} M R^2 + m_1 \cdot R^2 \Rightarrow \underline{I_O = 10\,kg \cdot m^2}$

② $\Rightarrow w_1 \cdot R = I_O \cdot a_{γων} \Rightarrow \boxed{a_{γων} = 1\,rad/s^2}$

2) $\Sigma\tau = I_O \cdot a'_{γων} \Rightarrow W_1 \cdot (OK) = I_O \cdot a'_{γων} \Rightarrow a'_{γων} = \frac{W_1 \cdot R συν60}{I_O} \Rightarrow \boxed{a'_{γων} = 0{,}5\,\frac{rad}{s^2}}$

Γ. Θα εφαρμόσουμε την αρχή διατήρησης της μηχανικής ενέργειας για την κίνηση του σώματος m_1 (μαζί με την τροχαλία) από την αρχική της θέση μέχρι το κατώτερο σημείο της τροχιάς του.



ΑΔΜΕ: $E_{ΜΗΧ}^{αρχ} = E_{ΜΗΧ}^{τελ} \Rightarrow K_{αρχ} + U_{αρχ} = K_{τελ} + U_{τελ} \Rightarrow$

$\Rightarrow m_1 g R + MgR = \frac{1}{2} I_O \omega^2 + MgR \Rightarrow \underline{\omega = \sqrt{2}\,\frac{rad}{s}}$

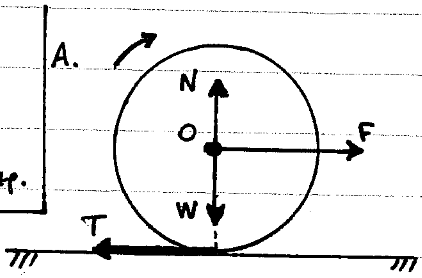
Επειδή για το σύστημα τροχαλία - μάζα m ισχύει $\Sigma\tau_{εξ} = 0$, μπορούμε να εφαρμόσουμε την αρχή διατήρησης της στροφορμής για την εκτίναξη του σώματος m.

Α.Δ.Στρ.: $\vec{L}_{αρχ} = \vec{L}_{τελ}$

$I_O \omega = I_{cm} \omega' + m υ R \Rightarrow \omega' = \frac{I_O \cdot \omega - m υ R}{I_{cm}} \Rightarrow \boxed{\omega' = 0}$

3.6 $m = 4\,kg$, $R = 0{,}5\,m$, $t_1 = 2\,s$, $N = \frac{10}{\pi}$ περ.

A.



Η σφαίρα κυλίεται χωρίς να ολισθαίνει εκτελώντας σύνθετη κίνηση:

- $\theta = N \cdot 2\pi \Rightarrow \underline{\theta = 20\,rad}$
- $\theta = \frac{1}{2} a_{γων} \cdot t^2 \Rightarrow a_{γων} = \frac{2 \cdot \theta}{t_1^2} \Rightarrow \underline{a_{γων} = 10\,\frac{rad}{s^2}}$
- $a_{cm} = a_{γων} \cdot R \Rightarrow \boxed{a_{cm} = 5\,m/s^2}$

Β. • $\Sigma\tau = I_{cm}\cdot a_{\gamma\omega\nu} \Rightarrow T\cdot R = \frac{2}{5}mR^2\cdot a_{\gamma\omega\nu} \Rightarrow T = \frac{2}{5}mR\,a_{\gamma\omega\nu} \Rightarrow \underline{T = 8N}$

• $\Sigma F = m a_{cm} \Rightarrow F - T = m a_{cm} \Rightarrow \underline{F = 28N}$

• $\left.\begin{array}{l} P_F = F\cdot u_{cm} \\ u_{cm} = a_{cm}t_1 = 10m/s \end{array}\right\} \Rightarrow \boxed{P_F = 280W}$

• $\left.\begin{array}{l} W_F = F\cdot x_{cm} \\ x_{cm} = \frac{1}{2}a_{cm}t_1^2 = 10m \end{array}\right\} \Rightarrow \boxed{W_F = 280\,J}$

Γ. • $\left.\begin{array}{l} P_T = \tau_T\cdot\omega = T\cdot R\cdot\omega \\ \omega = a_{\gamma\omega\nu}t_1 = 20\,rad/s \end{array}\right\} \Rightarrow \boxed{P_T = 80W}$ εκφράζει το ρυθμό μετατροπής ενός μέρους της ενέργειας που παίρνει το σώμα, σε στροφική κινητική ενέργεια.

• $\frac{K_{\sigma\tau\rho}}{K_{\mu\epsilon\tau}} = \frac{\frac{1}{2}I_{cm}\omega^2}{\frac{1}{2}m u_{cm}^2} = \frac{\frac{2}{5}mR^2\omega^2}{m u_{cm}^2} \xrightarrow{u_{cm}=\omega R} \boxed{\frac{K_{\sigma\tau\rho}}{K_{\mu\epsilon\tau}} = \frac{2}{5}}$

Δ. • Για να έχουμε κύλιση χωρίς ολίσθηση θα πρέπει $T < \mu_s\cdot N$ (1)

• $\Sigma\tau = I_{cm}\cdot a_{\gamma\omega\nu} \xrightarrow{a_{\gamma\omega\nu} = a_{cm}/R} TR = \frac{2}{5}mR^2\frac{a_{cm}}{R} \Rightarrow a_{cm} = \frac{5T}{2m}$ (2)

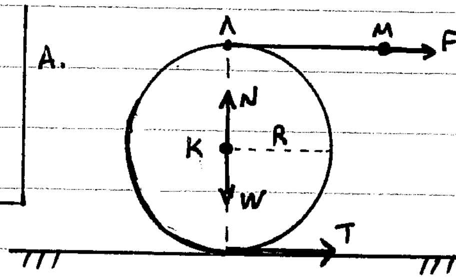
• $\Sigma F_x = m\cdot a_{cm} \xrightarrow{(2)} F - T = m\cdot\frac{5T}{2m} \Rightarrow F = \frac{7}{2}T \Rightarrow T = \frac{2}{7}F$ (3)

• $\Sigma F_y = 0 \Rightarrow N - w = 0 \Rightarrow N = mg = 40N$ (4)

(1) $\xrightarrow{(3),(4)} \frac{2}{7}F < 0{,}5\cdot 40 \Rightarrow \boxed{F < 70N}$

3.7 $m = 4kg$, $R = 0{,}2m$, $F = 12N$

A.



Ο τροχός κυλίεται χωρίς να ολισθαίνει εκτελώντας σύνθετη κίνηση:

• $\Sigma\tau = I_K\cdot a_{\gamma\omega\nu} \xrightarrow{a_{\gamma\omega\nu} = a_{cm}/R} FR - TR = \frac{1}{2}mR^2\frac{a_{cm}}{R}$
$\Rightarrow F - T = \frac{1}{2}m a_{cm} \Rightarrow \underline{T = F - \frac{m}{2}a_{cm}}$ (1)

• $\Sigma F = m a_{cm} \Longrightarrow F + T = m\cdot a_{cm} \xrightarrow{(1)} F + F - \frac{m}{2}a_{cm} = m a_{cm} \Rightarrow 2F = \frac{3m}{2}a_{cm}$
$\Rightarrow a_{cm} = \frac{4F}{3m} \Rightarrow \underline{a_{cm} = 4\,m/s^2}$

• $a_{\gamma\omega\nu} = \frac{a_{cm}}{R} \Rightarrow \boxed{a_{\gamma\omega\nu} = 20\,rad/s^2}$

• Το σημείο Μ έχει την ίδια επιτάχυνση με το σημείο Λ του τροχού.
$a_\Lambda = a_{cm} + a_\epsilon = a_{cm} + a_{\gamma\omega\nu}R = 2a_{cm}$ άρα $\boxed{a_M = 2a_{cm} = 8m/s^2}$

Β. • ① $\Rightarrow T = F - \frac{m}{2}\alpha_{cm} \Rightarrow T = (12 - \frac{4}{2}\cdot 4)\,N \Rightarrow \boxed{T = 4N}$

• $\Sigma F_y = 0 \Rightarrow N - W = 0 \Rightarrow \underline{N = mg = 40N}$

• Για να έχουμε κύλιση χωρίς ολίσθηση θα πρέπει:

$T < \mu \cdot N \Longrightarrow \mu > \frac{T}{N} \Rightarrow \mu > \frac{4}{40} \Rightarrow \boxed{\mu > 0,1}$

Γ. • $W_F = F\cdot x_{cm} + \tau_F \cdot \theta = F\cdot x_{cm} + F\cdot R\cdot \theta$

$x_{cm} = \frac{1}{2}\alpha_{cm} t_1^2 = 12,5m$

$\theta = \frac{1}{2}\alpha_{\gamma\omega\nu} t_1^2 = 62,5\,rad$

$\Rightarrow \boxed{W_F = 300\,J}$

2ος τρόπος: $W_F = F\cdot x_M$

$x_M = x_A = 2x_{cm} = 25m$

$\Rightarrow \underline{W_F = 300\,J}$

• $P_F = F\cdot U_{cm} + \tau_F \cdot \omega = F\cdot U_{cm} + F\cdot R\cdot \omega$

$U_{cm} = \alpha_{cm} t_1 = 10m/s$

$\omega = \alpha_{\gamma\omega\nu} t_1 = 50\,rad$

$\Rightarrow \boxed{P_F = 240W}$

2ος τρόπος: $P_F = F\cdot U_M$

$U_M = U_A = 2U_{cm} = 20m/s$

$\Rightarrow \underline{P_F = 240W}$

Δ. • $x_M = x_A = 2x_{cm} \Rightarrow \boxed{x_{cm} = 50m}$ x_{cm} = μετατόπιση τροχού

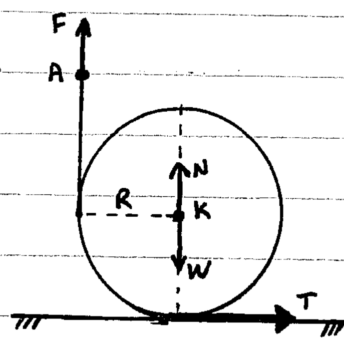
• Το μήκος (l) του σχοινιού που έχει ξετυλιχτεί αντιστοιχεί στο τόξο (S) που έχει διαγράψει ο τροχός στον οποίο είναι τυλιγμένο το σχοινί.

$\theta = \frac{S}{R} \Rightarrow S = \theta \cdot R \xrightarrow[x_{cm}=S]{S=l} \boxed{l = x_{cm} = 50m}$

3.8

$m = 4kg$
$R = 0,1m$
$F = 6N$

Α.



Ο τροχός κυλίεται χωρίς να ολισθαίνει, εκτελώντας σύνθετη κίνηση:

• $\Sigma\tau = I_K \cdot \alpha_{\gamma\omega\nu} \xrightarrow{\alpha_{\gamma\omega\nu} = \alpha_{cm}/R} F\cdot R - T R = \frac{1}{2} m R^2 \frac{\alpha_{cm}}{R}$

$\Rightarrow F - T = \frac{1}{2} m\alpha_{cm} \Rightarrow \underline{T = F - \frac{1}{2} m\alpha_{cm}}$ ①

• $\Sigma F_x = m\alpha_{cm} \Rightarrow T = m\alpha_{cm} \xrightarrow{①} F - \frac{1}{2}m\alpha_{cm} = m\alpha_{cm} \Rightarrow F = \frac{3}{2}m\alpha_{cm} \Rightarrow$

$\Rightarrow \alpha_{cm} = \frac{2F}{3m} \Rightarrow \underline{\alpha_{cm} = 1\,m/s^2}$

• $\frac{d\omega}{dt} = \alpha_{\gamma\omega\nu} = \frac{\alpha_{cm}}{R}$ ή $\boxed{\frac{d\omega}{dt} = 10\,rad/s^2}$

ΚΕΦΑΛΑΙΟ 3ο - ΘΕΜΑ 3ο

• $\frac{dL}{dt} = \Sigma\tau = FR - TR \xrightarrow{(1) \Rightarrow T=4N} \boxed{\frac{dL}{dt} = 0,2\ kgm^2/s^2}$

Β. • $K = \frac{1}{2} m u_{cm}^2 + \frac{1}{2} I_k \omega^2 \Rightarrow K = \frac{1}{2} m u_{cm}^2 + \frac{1}{2}\cdot\frac{1}{2} mR^2\omega^2 \xrightarrow{u_{cm}=\omega R}$

$\Rightarrow K = \frac{1}{2} m u_{cm}^2 + \frac{1}{4} m u_{cm}^2 \Rightarrow K = \frac{3}{4} m u_{cm}^2 \Rightarrow u_{cm} = \sqrt{\frac{4K}{3m}} \Rightarrow u_{cm} = 2\ m/s$

• $u_{cm} = a_{cm} t_1 \Rightarrow t_1 = 2s$

1) • $P_T = T\cdot u_{cm} - T\cdot R\cdot \omega$

$u_{cm} = a_{cm} t_1 = 2m/s$

$\omega = a_{\gamma\omega\nu} t_1 = 20\ rad$

$\Rightarrow P_T = (4\cdot 2 - 4\cdot 0,1\cdot 20)\ W \Rightarrow \boxed{P_T = 0}$

• $L = I\cdot\omega \Rightarrow L = \frac{1}{2} mR^2\cdot\omega \Rightarrow \boxed{L = 0,4\ kgm^2/s}$

2) • Το μήκος του σχοινιού (ℓ) που ξετυλίχτηκε είναι ίσο με το μήκος του τόξου που έχει διαγράψει ο τροχός στον οποίο είναι τυλιγμένο το σχοινί.

$\theta = \frac{s}{R} \Rightarrow s = \theta\cdot R$

$\theta = \frac{1}{2} a_{\gamma\omega\nu} t_1^2 \Rightarrow \theta = 20\ rad$

$\xrightarrow{s=\ell} \boxed{\ell = 2m}$

• $\bar{P}_F = \frac{W_F}{t_1}$

$W_F = \tau_F\cdot\theta = F\cdot R\cdot\theta = 12J$

$\Rightarrow \boxed{\bar{P}_F = 6W}$

Γ. • Για να έχουμε κύλιση χωρίς ολίσθηση θα πρέπει $T < \mu_s N$ (2)

• $\Sigma F_x = m\cdot a_{cm} \Rightarrow T = m a_{cm} \Rightarrow a_{cm} = \frac{T}{m}$ (3)

• $\Sigma F_y = 0 \Rightarrow N + F = w \Rightarrow N = w - F$ (4)

• $\Sigma\tau = I_k a_{\gamma\omega\nu} \Rightarrow F\cdot R - TR = \frac{1}{2} mR^2 \frac{a_{cm}}{R} \xrightarrow{(3)} F - T = \frac{1}{2} m\cdot\frac{T}{m} \Rightarrow$

$\Rightarrow \frac{3}{2} T = F \Rightarrow T = \frac{2F}{3}$ (5)

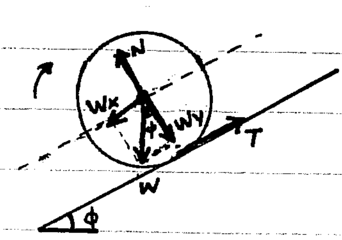
• (2) $\xrightarrow[(5)]{(4)} \frac{2F}{3} < \frac{1}{6}\cdot(w - F) \Rightarrow 4F < 40 - F \Rightarrow 5F < 40 \Rightarrow$

$\Rightarrow F < 8N$ ή $\boxed{F_{max} = 8N}$

3.9

$m = 10kg$
$R = 0,1m$
$\eta\mu\varphi = 0,56$
$U_0 = 8m/s$

Α.



• $\left|\frac{dL}{dt}\right| = |\Sigma\tau| = I_{cm}\cdot|a_{\gamma\omega\nu}|$ (1)

• $\Sigma\tau = I\cdot a_{\gamma\omega\nu} \Rightarrow -TR = \frac{2}{5} mR^2\cdot\frac{a_{cm}}{R} \Rightarrow$

$\Rightarrow T = -\frac{2}{5} m a_{cm}$ (2)

• $\Sigma F_x = m\cdot a_{cm} \Rightarrow T - W_x = m\cdot a_{cm} \xrightarrow[(2)]{W_x = W\eta\mu\varphi}$

$\Rightarrow -\frac{2}{5} m a_{cm} - W\eta\mu\varphi = m\cdot a_{cm} \Rightarrow -\frac{7}{5} m a_{cm} = mg\eta\mu\varphi \Rightarrow$

$\Rightarrow a_{cm} = -\frac{5}{7} g\eta\mu\varphi \Rightarrow a_{cm} = -4\ m/s^2$

• $a_{γων} = \frac{a_{cm}}{R} \Rightarrow a_{γων} = -40\ rad/s^2$ ή $|a_{γων}| = 40\ rad/s^2$

• ① $\Rightarrow \left|\frac{dL}{dt}\right| = \frac{2}{5} mR^2 |a_{γων}| \Rightarrow \boxed{\left|\frac{dL}{dt}\right| = 1,6\ kgm^2/s^2}$

Β. • $θ = N \cdot 2π \Rightarrow θ = 60\ rad$

• $θ = ω_0 t - \frac{1}{2}|a_{γων}|t^2 \xrightarrow{ω_0 = \frac{υ_0}{R} = 80\ rad/s} 60 = 80t - 20t^2 \Rightarrow t^2 - 4t + 3 = 0$

$Δ = β^2 - 4αγ \Rightarrow Δ = 4^2 - 4 \cdot 1 \cdot 3 \Rightarrow Δ = 4$

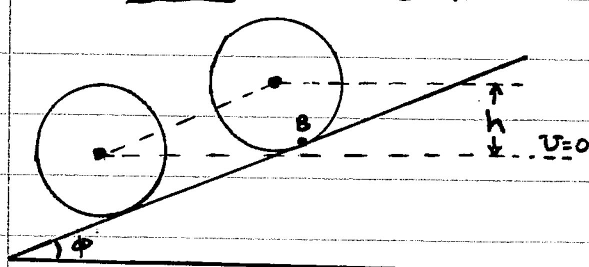
$t_{1,2} = \frac{-β \pm \sqrt{Δ}}{2α} \Rightarrow t_{1,2} = \frac{4 \pm 2}{2 \cdot 1} \Rightarrow \begin{cases} t_1 = 1s, & \text{(καθώς ανεβαίνει)} \\ t_2 = 3s, & \text{(καθώς κατεβαίνει).} \end{cases}$

• $υ_{cm} = υ_0 - |a_{cm}| \cdot t_1 \Rightarrow υ_{cm} = (8 - 4 \cdot 1)\ m/s \Rightarrow \boxed{υ_{cm} = 4\ m/s}$

Γ. • $S_{ολ} = \frac{υ_0^2}{|2 \cdot a_{cm}|} \Rightarrow S_{ολ} = \frac{8^2}{2 \cdot 4}\ m \Rightarrow \boxed{S_{ολ} = 8m}$

Δ.1) • Επειδή η τριβή είναι στατική (και το ολικό της έργο είναι μηδενικό), μπορούμε να εφαρμόσουμε αρχή διατήρησης της μηχανικής ενέργειας για την κάθοδο της σφαίρας.

ΑΔΜΕ: $E_{ΜΗΧ}^{αρχ} = E_{ΜΗΧ}^{τελ} \Rightarrow K_{αρχ} + U_{αρχ} = K_{τελ} + U_{τελ} \Rightarrow$



$\Rightarrow 0 + mgh_1 = \frac{1}{2} m υ_{cm}^2 + \frac{1}{2} I_{cm} ω^2 \Rightarrow$

$\Rightarrow mgh_1 = \frac{1}{2} m υ_{cm}^2 + \frac{1}{2} \frac{2}{5} mR^2 ω^2 \xrightarrow{υ_{cm} = ωR}$

$\Rightarrow mgh_1 = m υ_{cm}^2 \left(\frac{1}{2} + \frac{1}{5}\right) \Rightarrow$

$\Rightarrow gh_1 = \frac{7}{10} υ_{cm}^2 \Rightarrow υ_{cm} = \sqrt{\frac{10 \cdot g h_1}{7}} \Rightarrow$

$\Rightarrow \boxed{υ_{cm} = 5\ m/s}$

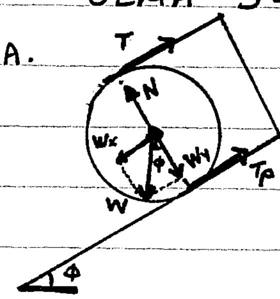
• $L = I_{cm} \cdot ω \xrightarrow{ω = \frac{υ_{cm}}{R} = 50\ rad/s} L = \frac{2}{5} mR^2 \cdot ω \Rightarrow \boxed{|L| = 2\ kgm^2/s}$

2) • Η κατακόρυφη μετατόπιση του κέντρου μάζας κατά h_1, αντιστοιχεί σε μετατόπιση του κυλίνδρου κατά μήκος του κεκλιμένου ίση με: $ημφ = \frac{h}{x_{cm}} \Rightarrow x_{cm} = π\ m$

• $x_{cm} = N \cdot 2πR \Rightarrow N = \frac{x_{cm}}{2πR} \Rightarrow \boxed{N = 5 \text{ περιστροφές}}$.

3.10 $\phi=30^\circ$, $w=20N$, $R=\frac{1}{3}m$

A.

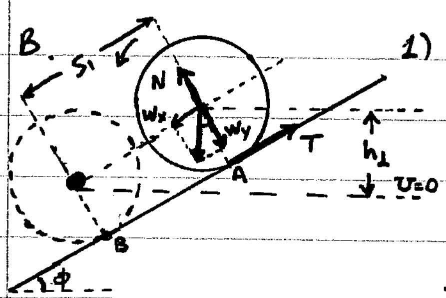


Ο κύλινδρος ισορροπεί:

- $\Sigma\tau=0 \Rightarrow T_p\cdot R - TR = 0 \Rightarrow \underline{T_p = T}$ ①
- $\Sigma F_x = 0 \Rightarrow T + T_p = W_x \xRightarrow{①} 2T_p = W\cdot\eta\mu\phi$
  $\Rightarrow \underline{T_p = 5N\ (=T)}$
- $\Sigma F_y = 0 \Rightarrow N - W_y = 0 \Rightarrow N = W\cdot\sigma\upsilon\nu\phi \Rightarrow$
  $\Rightarrow \underline{N = 10\sqrt{3}\,N}$

- Για να μην ολισθαίνει ο κύλινδρος θα πρέπει $T_p \leq \mu_s N \Rightarrow \mu_s > \frac{T_p}{N} \Rightarrow$
  $\Rightarrow \mu_s > \frac{5}{10\sqrt{3}} \Rightarrow \boxed{\mu_s > \frac{\sqrt{3}}{6}}$

B.



1) Ο κύλινδρος εκτελεί σύνθετη κίνηση:

- $\Sigma\tau = I\cdot a_{\gamma\omega\nu} \xRightarrow{a_{\gamma\omega\nu}=a_{cm}/R} TR = \frac{1}{2}MR^2\cdot\frac{a_{cm}}{R} \Rightarrow T = \frac{M\cdot a_{cm}}{2}$ ①
- $\Sigma F = M\cdot a_{cm} \Rightarrow W_x - T = M a_{cm} \xRightarrow{①} W_x - \frac{Ma_{cm}}{2} = Ma_{cm} \Rightarrow$
  $\Rightarrow W\cdot\eta\mu\phi = \frac{3}{2}M\cdot a_{cm} \Rightarrow a_{cm} = \frac{2}{3}g\cdot\eta\mu\phi \Rightarrow \underline{a_{cm} = \frac{10}{3}m/s^2}$
- $a_{\gamma\omega\nu} = \frac{a_{cm}}{R} \Rightarrow \boxed{a_{\gamma\omega\nu} = 10\ rad/s^2}$

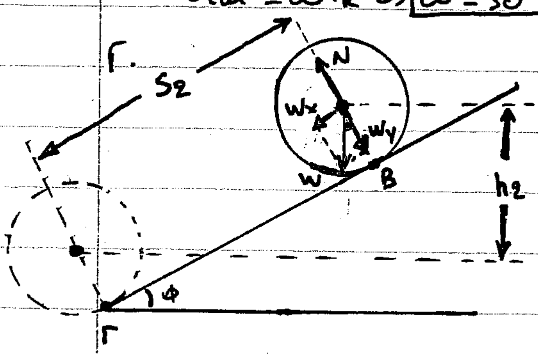
2) Εφαρμόζουμε Α.Δ.Μ.Ε. για την κίνηση του κυλίνδρου (αφού $W_T=0$):

$E_{MHX}^{\alpha\rho\chi} = E_{MHX}^{\tau\epsilon\lambda} \Rightarrow K_{\alpha\rho\chi} + U_{\alpha\rho\chi} = K_{\tau\epsilon\lambda} + U_{\tau\epsilon\lambda} \Rightarrow Mgh_1 = \frac{1}{2}MU_{cm}^2 + \frac{1}{2}I\omega^2 \Rightarrow$

$\Rightarrow Mgh_1 = \frac{1}{2}MU_{cm}^2 + \frac{1}{4}MR^2\omega^2 \xRightarrow{U_{cm}=\omega R} Mgh_1 = \frac{3}{4}MU_{cm}^2 \Rightarrow \underline{U_{cm} = 10m/s}$

- $U_{cm} = \omega\cdot R \Rightarrow \boxed{\omega = 30\ rad/s}$

Γ.



Απο το σημείο Β ως το σημείο Γ, επειδή δεν υπάρχει τριβή ο κύλινδρος κυλίεται και ολισθαίνει (ισχύει $U_{cm} \neq \omega R$, $a_{cm} \neq a_{\gamma\omega\nu}R$). Συγκεκριμένα ο κύλινδρος από μεταφορική σκοπιά εκτελεί επιταχυνόμενη κίνηση ενώ από στροφική σκοπιά ομαλή κυκλική.

1) Εφαρμόζουμε Α.Δ.Μ.Ε. για την κίνηση του κυλίνδρου:

$E_{MHX}^{\alpha\rho\chi} = E_{MHX}^{\tau\epsilon\lambda} \Rightarrow K_{\alpha\rho\chi} + U_{\alpha\rho\chi} = K_{\tau\epsilon\lambda} + U_{\tau\epsilon\lambda} \Rightarrow \frac{1}{2}MU_{cm}^2 + \frac{1}{2}I\omega^2 + Mgh_2 = K_{\tau\epsilon\lambda} \Rightarrow$

$\Rightarrow \frac{1}{2}MU_{cm}^2 + \frac{1}{4}MR^2\omega^2 + Mgh_2 = K_{\tau\epsilon\lambda} \Rightarrow \frac{3}{4}MU_{cm}^2 + Mgh_2 = K_{\tau\epsilon\lambda} \xRightarrow{W=Mg\Rightarrow M=2kg}$

$\Rightarrow h_2 = \frac{450-150}{20}m \Rightarrow \underline{h_2 = 15m}$

$$\left.\begin{aligned} &\eta\mu\phi = \frac{h_1}{S_1} \Rightarrow S_1 = 15m \\ &\eta\mu\phi = \frac{h_2}{S_2} \Rightarrow S_2 = 30m \end{aligned}\right\} \Rightarrow S_{o\lambda} = S_1 + S_2 \Rightarrow \boxed{S_{o\lambda} = 45m}$$

2) ■ Κίνηση από το Α στο Β: · $S_1 = \frac{1}{2}\alpha_{cm} t_1^2 \Rightarrow t_1 = \sqrt{\frac{2S_1}{\alpha_{cm}}} \Rightarrow \underline{t_1 = 3s}$

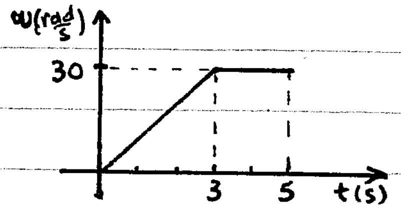
■ Κίνηση από το Β στο Γ: · $\Sigma F_x = M\cdot\alpha'_{cm} \Rightarrow W_x = M\cdot\alpha'_{cm} \Rightarrow W\eta\mu\phi = M\alpha'_{cm} \Rightarrow$

$\Rightarrow \underline{\alpha'_{cm} = 5\,m/s^2}$

· $K_{\tau\varepsilon\lambda} = \frac{1}{2}MU'^2_{cm} + \frac{1}{2}I\omega^2 \xRightarrow{\omega=30\,rad/s} 450 = \frac{1}{2}\cdot 2\cdot U'^2_{cm} + \frac{1}{2}\cdot\frac{1}{2}\cdot 2\cdot\frac{1}{9}\cdot 900 \Rightarrow$

$\Rightarrow \underline{U'_{cm} = 20\,m/s}$

· $U'_{cm} = U_0 + \alpha'_{cm} t \xRightarrow{U_0 = 10\,m/s} 20 = 10 + 5t_2 \Rightarrow \underline{t_2 = 2s}$



· $t \leq 3s$: $\omega = \alpha_{\gamma\omega\nu} t \Rightarrow \underline{\omega = 10\cdot t}$

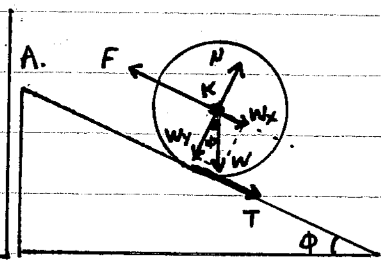
· $t > 3s$: $\omega = 30\,rad/s$ (= σταθερή).

■ $\theta_{o\lambda} = \text{Εμβαδό}_{(\tau\rho\alpha\pi\varepsilon\zeta\acute{\iota}o\upsilon)} = \frac{5+2}{2}\cdot 30$ ή $\theta_{o\lambda} = 105\,rad$

$\theta_{o\lambda} = N_{o\lambda}\cdot 2\pi \Rightarrow \boxed{N_{o\lambda} = \frac{52,5}{\pi} \text{ περιστροφές}}$.

3.11 $R = 0,2m$, $M = 2kg$, $\phi = 30^\circ$, $F = 25N$

A.



Ο κύλινδρος εκτελεί σύνθετη κίνηση:

· $\Sigma\tau = I_K\cdot\alpha_{\gamma\omega\nu} \xRightarrow{\alpha_{\gamma\omega\nu} = \alpha_{cm}/R} T\cdot R = \frac{1}{2}MR^2\frac{\alpha_{cm}}{R} \Rightarrow$

$\Rightarrow T = \frac{1}{2}M\alpha_{cm}$ (1)

· $\Sigma F_x = M\alpha_{cm} \Rightarrow F - T - W_x = M\cdot\alpha_{cm} \xRightarrow{(1)}$

$\Rightarrow F - \frac{1}{2}M\alpha_{cm} - W\eta\mu\phi = M\cdot\alpha_{cm} \Rightarrow F - Mg\eta\mu\phi = \frac{3}{2}M\alpha_{cm} \Rightarrow \boxed{\alpha_{cm} = 5\,m/s^2}$

B. · (1) $\Rightarrow \boxed{T = 5N}$

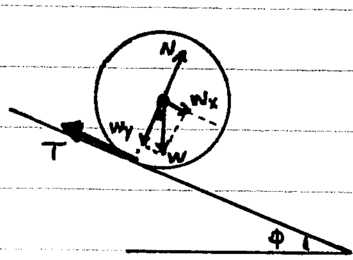
· $\frac{K_{\mu\varepsilon\tau}}{K_{\sigma\tau\rho}} = \frac{\frac{1}{2}MU_{cm}^2}{\frac{1}{2}I\omega^2} = \frac{MU_{cm}^2}{\frac{1}{2}MR^2\omega^2} \xRightarrow{U_{cm} = \omega R} \frac{MU_{cm}^2}{\frac{1}{2}MU_{cm}^2}$ ή $\boxed{\frac{K_{\mu\varepsilon\tau}}{K_{\sigma\tau\rho}} = 2}$

Γ. · $\omega_1 = \alpha_{\gamma\omega\nu} t_1 \xRightarrow{\alpha_{\gamma\omega\nu} = \frac{\alpha_{cm}}{R} = 25\,rad/s^2} \underline{\omega_1 = 50\,rad/s}$

· $L = I\cdot\omega_1 \Rightarrow L = \frac{1}{2}MR^2\omega_1 \Rightarrow \boxed{L = 2\,kgm^2/s}$

· $S_1 = \frac{1}{2}\alpha_{cm} t_1^2 \Rightarrow \boxed{S_1 = 10m}$

Δ.



· $\Sigma\tau = I\cdot\alpha'_{\gamma\omega\nu} \xRightarrow{\alpha'_{\gamma\omega\nu} = \alpha'_{cm}/R} -T\cdot R = \frac{1}{2}MR^2\frac{\alpha'_{cm}}{R} \Rightarrow$

$\Rightarrow T = -\frac{1}{2}M\alpha'_{cm}$ (2)

· $\Sigma F_x = M\alpha'_{cm} \Rightarrow T - W_x = M\alpha'_{cm} \xRightarrow{(2)}$

$\Rightarrow -\frac{1}{2}M\alpha'_{cm} - Mg\eta\mu\phi = M\alpha'_{cm} \Rightarrow Mg\eta\mu\phi = \frac{3}{2}M\alpha'_{cm} \Rightarrow$

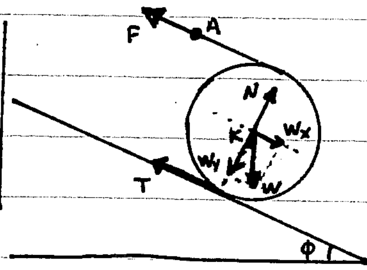
$\Rightarrow \alpha'_{cm} = -\frac{2}{3}g\eta\mu\phi \Rightarrow \underline{\alpha'_{cm} = -\frac{10}{3}\,m/s^2}$

• $S_2 = \frac{u_0^2}{2|a_{cm}'|} \xrightarrow{u_0 = \omega_2 \cdot R = 10 m/s} S_2 = \frac{100}{\frac{20}{3}} m \Rightarrow \underline{S_2 = 15m}$

• $S_{ολ} = S_1 + S_2 \Rightarrow \underline{S_{ολ} = 25m}$

• $ημφ = \frac{h}{S_{ολ}} \Rightarrow \boxed{h = 12,5m}$

3.12 $M = 3kg$, $R = 0,3m$, $φ = 30°$



Α. Ο κύλινδρος ισορροπεί:

$\cdot \Sigma\tau_{(K)} = 0 \Rightarrow FR - TR = 0 \Rightarrow F = T$ (1)

$\cdot \Sigma F_x = 0 \Rightarrow F + T - W_x = 0 \xrightarrow{(1)} 2F = Mgημ30° \Rightarrow$

$\Rightarrow \boxed{F = 7,5N}$

(1) $\Rightarrow \boxed{T = 7,5N}$

Β. Ο κύλινδρος κυλίεται χωρίς να ολισθαίνει εκτελώντας σύνθετη κίνηση:

$\Sigma\tau = I_{cm} \cdot a_{γων} \xrightarrow{a_{γων} = a_{cm}/R} FR - T'R = \frac{1}{2} MR^2 \frac{a_{cm}}{R} \Rightarrow T' = F - \frac{Ma_{cm}}{2}$ (2)

$\Sigma F_x = Ma_{cm} \Rightarrow F + T' - W_x = Ma_{cm} \xrightarrow{(2)} F + F - \frac{Ma_{cm}}{2} - Mgημφ = Ma_{cm} \Rightarrow$

$\Rightarrow 2F - Mgημφ = \frac{3}{2} Ma_{cm} \Rightarrow \underline{a_{cm} = \frac{10}{3} m/s^2}$

(2) $\Rightarrow \boxed{T' = 10N}$

Γ. $\cdot\ x_{cm} = \frac{1}{2} a_{cm} t_1^2 \Rightarrow \boxed{x_{cm} = 15m}$

$\cdot\ x_A = 2x_{cm} \Rightarrow \boxed{x_A = 30m}$

$\left.\begin{array}{l} \cdot\ \ell = S = \theta \cdot R \\ \theta = \frac{1}{2} a_{γων} t_1^2 \xrightarrow{a_{γων} = a_{cm}/R = \frac{100}{9} rad/s^2} \theta = 50 rad \end{array}\right\} \Rightarrow \boxed{\ell = 15m}$

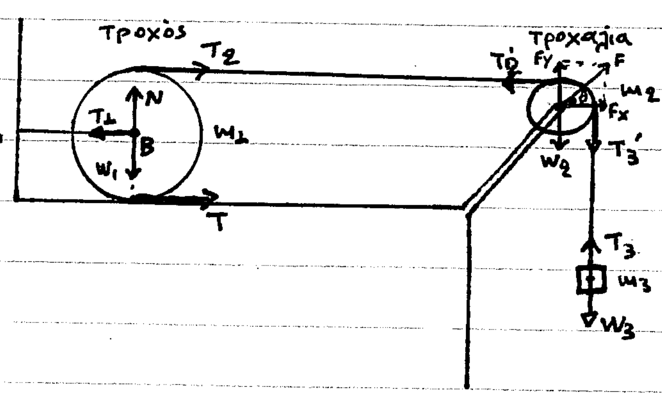
Δ. $\cdot\ W_F = F \cdot x_{cm} + \tau_F \cdot \theta \Rightarrow W_F = F \cdot x_{cm} + F \cdot R\theta \Rightarrow \boxed{W_F = 450J}$

$\left.\begin{array}{l} \cdot\ P_F = F \cdot u_{cm} + \tau_F \cdot \omega \Rightarrow P_F = F \cdot u_{cm} + F \cdot R\omega \xrightarrow{u_{cm} = \omega R} P_F = 2F u_{cm} \\ u_{cm} = a_{cm} t_1 \Rightarrow u_{cm} = 10 m/s \end{array}\right\} \Rightarrow \boxed{P_F = 300W}$

3.13 Τροχός: $m_1 = 6kg$, $R_1 = 0,25m$

Τροχαλία: $m_2 = 1kg$, $R_2 = 0,1m$

Σώμα: $m_3 = 3kg$



Α. Το σύστημα ισορροπεί:

• Σώμα m_3: $\Sigma F=0 \Rightarrow T_3-w_3=0 \Rightarrow T_3=m_3 g \Rightarrow \underline{T_3=30N\ (=T_3')}$

• Τροχαλία: $\Sigma\tau=0 \Rightarrow T_2'R-T_3'R=0 \Rightarrow \underline{T_2'=T_3'=30N\ (=T_2)}$

$\Sigma F_x=0 \Rightarrow F_x-T_2'=0 \Rightarrow \underline{F_x=30N}$

$\Sigma F_y=0 \Rightarrow F_y-w_2-T_3'=0 \Rightarrow F_y=m_2 g+T_3' \Rightarrow \underline{F_y=40N}$

$F=\sqrt{F_x^2+F_y^2} \Rightarrow \boxed{F=50N}$

$\varepsilon\varphi\theta=\frac{F_y}{F_x}=\frac{4}{3}$

• Τροχός: $\Sigma\tau=0 \Rightarrow TR-T_2R=0 \Rightarrow \underline{T=T_2=30N}$

$\Sigma F_x=0 \Rightarrow T_1-T-T_2=0 \Rightarrow \boxed{T_1=60N}$

Β. • Σώμα m_3: $\Sigma F=0 \Rightarrow w_3-T_3=m_3\alpha_{cm} \xRightarrow{S.I.} \underline{T_3=30-3\alpha_{cm}}$ ①

• Τροχαλία: $\Sigma\tau=I\cdot\alpha_{\gamma\omega\nu} \xRightarrow{\alpha_{\gamma\omega\nu}=\alpha_{cm}/R} T_3'R_2-T_2'R_2=\frac{1}{2}m_2R_2^2\frac{\alpha_{cm}}{R_2} \xRightarrow[S.I.]{①}$

$\Rightarrow 30-3\alpha_{cm}-T_2'=\frac{\alpha_{cm}}{2} \Rightarrow \underline{T_2'=30-3{,}5\alpha_{cm}\ (=T_2)}$ ②

• Τροχός: $\Sigma\tau=I\alpha_{\gamma\omega\nu}' \xRightarrow{\alpha_{\gamma\omega\nu}=\alpha_{cm}/R} T_2R_1-TR_1=\frac{1}{2}m_1R_1^2\frac{\alpha_{cm}}{R_1} \xRightarrow{②}$

$\Rightarrow 30-3{,}5\alpha_{cm}-4=3\alpha_{cm} \Rightarrow 26=6{,}5\alpha_{cm} \Rightarrow \boxed{\alpha_{cm}=4m/s^2}$

Γ. • Τροχαλία: $\theta=N\cdot2\pi \Rightarrow \theta=20rad$

$\left.\begin{array}{l}\theta=\frac{1}{2}\alpha_{\gamma\omega\nu}t^2\\ \alpha_{\gamma\omega\nu}=\frac{\alpha_{cm}}{R_2}=40rad/s^2\end{array}\right\} \Rightarrow \underline{t_1=1s}$

1) $\left.\begin{array}{l}K_2=\frac{1}{2}I\omega^2\\ \omega=\alpha_{\gamma\omega\nu}t_1=40rad/s\end{array}\right\} \Rightarrow K_2=\frac{1}{2}\frac{1}{2}1\cdot0{,}01\cdot1600J \Rightarrow \underline{K_2=4J}$

2) • $\left.\begin{array}{l}K_3=\frac{1}{2}m_3\upsilon^2\\ \upsilon=\alpha_{cm}t_1=4m/s\end{array}\right\} \Rightarrow \underline{K_3=24J}$

• $\left.\begin{array}{l}W_{w_3}=w_3\cdot y\\ y=\frac{1}{2}\alpha_{cm}t_1^2=2m\end{array}\right\} \Rightarrow \underline{W_{w_3}=60J}$

3) • $\left.\begin{array}{l}K_1=\frac{1}{2}I\omega^2\\ \left.\begin{array}{l}\omega=\alpha_{\gamma\omega\nu}'t_1\\ \alpha_{\gamma\omega\nu}'=\frac{\alpha_{cm}}{R_1}=16rad/s^2\end{array}\right\}\Rightarrow\omega=16rad/s\end{array}\right\} \Rightarrow K_1=\frac{1}{2}\cdot\frac{1}{2}6\cdot\frac{1}{16}\cdot16^2J \Rightarrow$

$\Rightarrow \underline{K_1=24J}$

• $\left.\begin{array}{l}W_T=\tau_T\cdot\theta=-T\cdot R_1\theta\\ \theta=\frac{1}{2}\alpha_{\gamma\omega\nu}'t_1^2=8rad\end{array}\right\} \Rightarrow \underline{W_T=-8J}$

Η αρχή διατήρησης της ενέργειας είναι:

Α.Δ.Ε. : $W_{w_3} = K_3 + K_2 + K_1 + E_{απωλειών} \xrightarrow{E_{απωλ} = Q = |W_T|}$

$$W_{w_3} = K_3 + K_2 + K_1 + |W_T| \quad (3)$$

Αν αντικαταστήσουμε τις τιμές που βρήκαμε στη σχέση (3) διαπιστώνουμε ότι ισχύει η αρχή διατήρησης της ενέργειας.

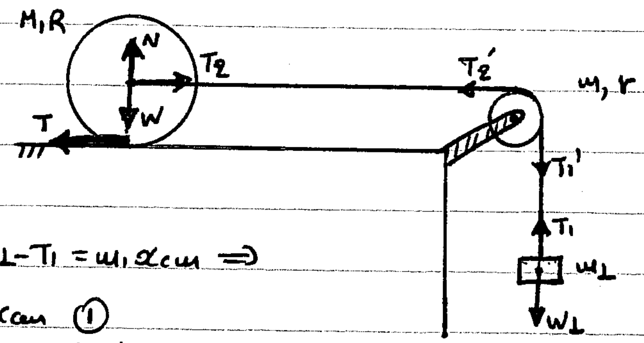
Δ. • (1) $\Rightarrow T_3 = 18\,N \; (=T_3')$

$P_{T_3'} = \tau_{T_3'} \cdot \omega = T_3' \cdot R_2 \cdot \omega$ ή $\boxed{P_{T_3'} = 72\,W}$

• (2) $\Rightarrow T_2' = 16\,N \; (=T_2)$

$P_{T_2'} = \tau_{T_2'}\,\omega = -T_2' \cdot R_2\,\omega$ ή $\boxed{P_{T_2'} = -64\,W}$

3.14 Τροχαλία: $m = 2kg$, $r = 0,1m$
Σώμα $m_1 = 3kg$
Τροχός: $M = 4kg$, $R = 0,3m$



Α. • Σώμα m_1: $\Sigma F = m_1 \alpha_{cm} \Rightarrow W_1 - T_1 = m_1 \alpha_{cm} \Rightarrow$

$\xRightarrow{S.I.} T_1 = 30 - 3\alpha_{cm}$ (1)

• Τροχαλία: $\Sigma\tau = I\alpha_{γων} \xrightarrow{\alpha_{γων} = \alpha_{cm}/R} T_1' r - T_2' r = \frac{1}{2} m r^2 \frac{\alpha_{cm}}{r} \xrightarrow[(1)]{T_1' = T_1,\; T_2' = T_2}$

$\Rightarrow 30 - 3\alpha_{cm} - T_2 = \alpha_{cm} \Rightarrow T_2 = 30 - 4\alpha_{cm}$ (2)

• Τροχός: • $\Sigma\tau = I \cdot \alpha_{γων} \xrightarrow{\alpha_{γων} = \alpha_{cm}/R} T \cdot R = \frac{1}{2} M R^2 \frac{\alpha_{cm}}{R} \xRightarrow{S.I.} T = 2\alpha_{cm}$ (3)

• $\Sigma F_x = M\alpha_{cm} \Rightarrow T_2 - T = M\alpha_{cm} \xrightarrow[S.I.]{(2),(3)} 30 - 4\alpha_{cm} - 2\alpha_{cm} = 4\alpha_{cm} \Rightarrow$

$\Rightarrow \boxed{\alpha_{cm} = 3\,m/s^2}$

Β. $\left.\begin{array}{l} \frac{dL_{τροχαλίας}}{dt} = \Sigma\tau = I\alpha_{γων} = \frac{1}{2} m r^2 \alpha_{γων} \\ \alpha_{γων} = \frac{\alpha_{cm}}{r} = 30\,rad/s^2 \end{array}\right\} \Rightarrow \boxed{\frac{dL_{τροχ.}}{dt} = 0,3\,kg m^2/s^2}$

Γ. • Τροχαλία: $\theta = \frac{1}{2}\alpha_{γων} t_1^2 \Rightarrow \underline{t_1 = 2s}$

• Τροχός: $\left.\begin{array}{l} L = I\omega = \frac{1}{2} M R^2 \omega \\ \left.\begin{array}{l} \omega = \alpha'_{γων} t_1 \\ \alpha'_{γων} = \frac{\alpha_{cm}}{R} = 10\,rad/s^2 \end{array}\right\} \Rightarrow \omega = 20\,rad/s \end{array}\right\} \Rightarrow \boxed{L = 3,6\,kg m^2/s}$

ΚΕΦΑΛΑΙΟ 3ο - ΘΕΜΑ 3ο

• Σώμα m_1: $y = \frac{1}{2}\alpha_{cm} t_1^2 \Rightarrow \boxed{y_1 = 6m}$

Δ. $\Pi\% = \frac{K_{τροχού}}{W_{w_1}} \cdot 100\%$ (4)

• $K_{τροχού} = \frac{1}{2} M v_{cm}^2 + \frac{1}{2} I \omega^2 = \frac{1}{2} M v_{cm}^2 + \frac{1}{2}\frac{1}{2} M R^2 \omega^2 \xrightarrow{v_{cm}=\omega R} \frac{1}{2} M v_{cm}^2 + \frac{1}{4} M v_{cm}^2$

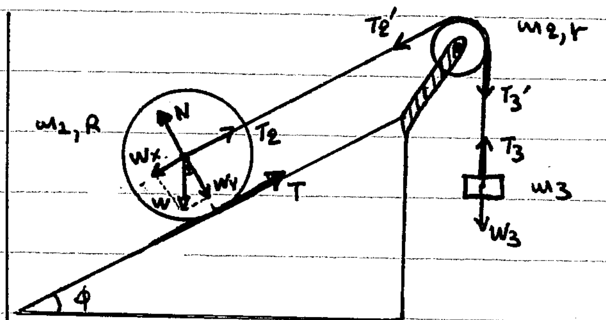
$\left.\begin{array}{l} \Rightarrow K_{τροχού} = \frac{3}{4} M v_{cm}^2 \\ v_{cm} = \alpha_{cm} t_1 = 6m/s \end{array}\right\} \Rightarrow K_{τροχού} = 108J$

• $W_{w_1} = w_1 \cdot y_1 = m_1 g y_1$ ή $W_{w_1} = 180J$

(4) $\Rightarrow \Pi\% = \frac{108}{180} \cdot 100\% \Rightarrow \boxed{\Pi\% = 60\%}$

3.15

$m_3 = 1kg$
τροχός: $m_1 = 4kg$, $R = 0{,}5m$
τροχαλία: $m_2 = 2kg$, $r = 0{,}1m$
$\alpha_{cm} = 1{,}25 \, m/s^2$
$I_1 = 0{,}5 \, kgm^2$, $\phi = 30°$



• Ως προς τον άξονα περιστροφής της τροχαλίας:

$\tau_{w_x} = m_1 g ημ\phi \, r = 2Nm$

$\tau_{w_3} = m_3 g r = 1Nm$

Επειδή $\tau_{w_x} > \tau_{w_3}$ το m_3 θα κινηθεί προς τα πάνω.

A. • Σώμα m_3: $\Sigma F = m_3 \alpha_{cm} \Rightarrow T_3 - W_3 = m_3 \alpha_{cm} \Rightarrow T_3 = m_3 g + m_3 \alpha_{cm} \Rightarrow$

$\Rightarrow T_3 = 11{,}25N \; (=T_3')$

• Τροχός: $\Sigma\tau = I \alpha_{γων} \xrightarrow{\alpha_{γων} = \alpha_{cm}/R} T \cdot R = I_1 \cdot \frac{\alpha_{cm}}{R} \Rightarrow T = \frac{0{,}5 \cdot 1{,}25}{0{,}25} N \Rightarrow$

$\Rightarrow T = 2{,}5N$

$\Sigma F_x = m_1 \alpha_{cm} \Rightarrow w_x - T_2 - T = m_1 \alpha_{cm} \Rightarrow T_2 = m_1 g ημ\phi - T - m_1 \alpha_{cm} \Rightarrow$

$\Rightarrow T_2 = 12{,}5N \; (=T_2')$

• Τροχαλία: $\Sigma\tau = I \cdot \alpha_{γων} \xrightarrow{\alpha_{γων} = \alpha_{cm}/R} T_2' r - T_1' r = I_2 \cdot \frac{\alpha_{cm}}{r} \Rightarrow$

$\Rightarrow I_2 = \frac{(T_2' - T_1') r^2}{\alpha_{cm}} \Rightarrow \boxed{I_2 = 0{,}01 \, kg \cdot m^2}$

B. $\frac{dL_{τροχαλίας}}{dt} = \Sigma\tau = I_2 \cdot \alpha_{γων} = I_2 \cdot \frac{\alpha_{cm}}{r} = 0{,}01 \cdot \frac{1{,}25}{0{,}1} \, kgm^2/s^2$

ή $\boxed{\frac{dL_{τροχαλίας}}{dt} = 0{,}125 \, kgm^2/s^2}$

Γ. · Τροχαλία : $L_2 = I_2 \cdot \omega_1 \Rightarrow \omega_1 = 50\ rad/s$

$$\left.\begin{array}{l} \omega_1 = a_{γων} t_1 \\ a_{γων} = \frac{a_{cm}}{r} = 12{,}5\ rad/s^2 \end{array}\right\} \Rightarrow \underline{t_1 = 4s}$$

· Τροχός : $L_1 = I_1 \omega_1'$

$$\left.\begin{array}{l} L_1 = I_1 \omega_1' \\ \left.\begin{array}{l} \omega_1' = a_{γων}' t_1 \\ a_{γων}' = \frac{a_{cm}}{R} = 2{,}5\ rad/s^2 \end{array}\right\} \Rightarrow \omega_1' = 10\ rad/s \end{array}\right\} \Rightarrow \boxed{L_1 = 5\ kg m^2/s}$$

Δ. $$\left.\begin{array}{l} W_W = W_x \cdot x = m_1 g \eta \mu \varphi \cdot x \\ x = \frac{1}{2} a_{cm} t_1^2 \Rightarrow x = 10m \end{array}\right\} \Rightarrow W_W = 200J.$$

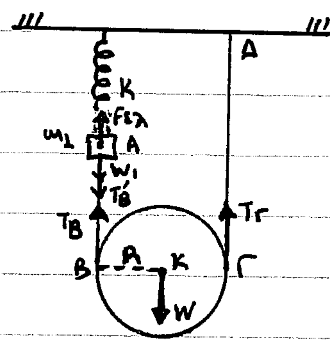
1) $$\left.\begin{array}{l} \Pi 1\% = \frac{K_1}{W_W} \cdot 100\% \\ \left.\begin{array}{l} K_1 = \frac{1}{2} m_1 U_{cm}^2 + \frac{1}{2} I_1 \omega_1'^2 \\ U_{cm} = a_{cm} t_1 = 5m/s \end{array}\right\} \Rightarrow \underline{K_1 = 75J} \end{array}\right\} \Rightarrow \Pi 1\% = \frac{75}{200} 100\% \Rightarrow \boxed{\Pi 1\% = 37{,}5\%}$$

2) $$\left.\begin{array}{l} \Pi 2\% = \frac{K_2}{W_W} \cdot 100\% \\ K_2 = \frac{1}{2} I_2 \omega_1^2 \Rightarrow \underline{K_2 = 12{,}5J} \end{array}\right\} \Rightarrow \Pi 2\% = \frac{12{,}5}{200} 100\% \Rightarrow \boxed{\Pi 2\% = 6{,}25\%}$$

3) $$\left.\begin{array}{l} \Pi 3\% = \frac{K_3}{W_W} \cdot 100\% \\ K_3 = \frac{1}{2} m_3 U_{cm}^2 \Rightarrow \underline{K_3 = 12{,}5J} \end{array}\right\} \Rightarrow \Pi 3\% = \frac{12{,}5}{200} 100\% \Rightarrow \boxed{\Pi 3\% = 6{,}25\%}$$

4) $$\left.\begin{array}{l} \Pi 4\% = \frac{\Delta U_3}{W_W} \cdot 100\% \\ \left.\begin{array}{l} \Delta U_3 = m_3 g y. \\ y = \frac{1}{2} a_{cm} t_1^2 = 10m \end{array}\right\} \Rightarrow \Delta U_3 = 100J \end{array}\right\} \Rightarrow \Pi 4\% = \frac{100}{200} 100\% \Rightarrow \boxed{\Pi 4\% = 50\%}$$

3.16 $m_1 = 1\ kg$
$k = 100\ N/m$
$M = 4kg$, $R = 0{,}3m$



A. Ο δίσκος ισορροπεί:

$\cdot \Sigma \tau_{(K)} = 0 \Rightarrow T_Γ \cdot R - T_B \cdot R = 0 \Rightarrow$

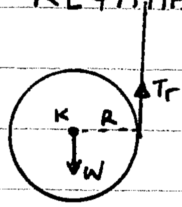
$\Rightarrow T_Γ = T_B$ (1)

$\cdot \Sigma F = 0 \Rightarrow T_B + T_Γ - W = 0 \xRightarrow{(1)}$

$2T_Γ = Mg \Rightarrow \underline{T_Γ = 20N\ (= T_B)}$

B. Ο δίσκος εκτελεί σύνθετη κίνηση.

$$\begin{cases} U_{cm} = a_{cm} t \\ h = \frac{1}{2} a_{cm} t^2 \end{cases} \Rightarrow \begin{cases} t = \frac{U_{cm}}{a_{cm}} \\ h = \frac{1}{2} a_{cm} \frac{U_{cm}^2}{a_{cm}^2} \end{cases} \Rightarrow \begin{cases} t = 0{,}3s \\ a_{cm} = \frac{U_{cm}^2}{2h} = \frac{20}{3} m/s^2 \end{cases}$$



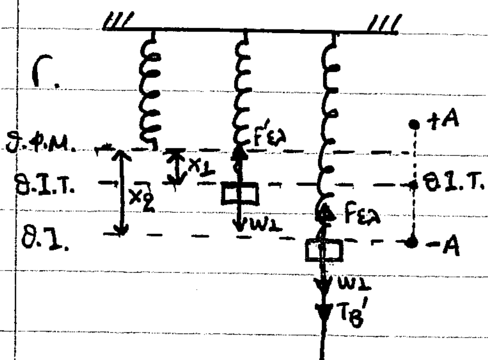
• $\Sigma\tau = I_K \cdot a_{\gamma\omega\nu} \xrightarrow{a_{\gamma\omega\nu}=a_{cm}/R} T_r \cdot R = I_K \cdot \frac{a_{cm}}{R} \Rightarrow I_K = \frac{T_r \cdot R^2}{a_{cm}}$ (2)

• $\Sigma F = M a_{cm} \Rightarrow W - T_r = M a_{cm} \Rightarrow T_r = Mg - M a_{cm} \Rightarrow \underline{T_r = \frac{40}{3} N}$

(2) $\Rightarrow I_K = \dfrac{\frac{40}{3} \cdot 0{,}09}{\frac{20}{3}}\ kg\,m^2 \Rightarrow \underline{I_K = 0{,}18\ kg\,m^2}$

$$\frac{K_{\sigma\tau\rho}}{K_{\mu\varepsilon\tau}} = \frac{\frac{1}{2} I_K \omega^2}{\frac{1}{2} M v_{cm}^2} \xrightarrow{\omega = v_{cm}/R = \frac{20}{3} rad/s} \frac{K_{\sigma\tau\rho}}{K_{\mu\varepsilon\tau}} = \frac{0{,}18 \cdot \frac{400}{9}}{4 \cdot 4} \Rightarrow \boxed{\frac{K_{\sigma\tau\rho}}{K_{\mu\varepsilon\tau}} = \frac{1}{2}}$$

Γ.



• <u>Θ.Ι.</u> (πριν κοπεί το νήμα ΑΒ):

$\Sigma F = 0 \Rightarrow F_{\varepsilon\lambda} - W_1 - T_B' = 0 \xrightarrow{T_B' = T_B = 20N} k \cdot x_2 = m_1 g + T_B'$

$\Rightarrow \underline{x_2 = 0{,}3 m}$

• <u>Θ.Ι.Τ</u>

$\Sigma F = 0 \Rightarrow F'_{\varepsilon\lambda} - W_1 = 0 \Rightarrow k \cdot x_1 = m_1 g \Rightarrow \underline{x_1 = 0{,}1 m}$

• $A = x_2 - x_1 \Rightarrow \underline{A = 0{,}2 m}$
• $\omega = \sqrt{\frac{k}{m_1}} \Rightarrow \underline{\omega = 10\ rad/s}$
• $t_0 = 0,\ x = -A$.

$x = A\eta\mu(\omega t + \varphi_0) \xrightarrow[x=-A]{t_0=0} -A = A\eta\mu\varphi_0 \Rightarrow \eta\mu\varphi_0 = -1 = \eta\mu\frac{3\pi}{2}$ ή $\underline{\varphi_0 = \frac{3\pi}{2} rad}$

Άρα $\boxed{x = 0{,}2\eta\mu(10t + \frac{3\pi}{2})}$, (S.I.)

Δ. • $F_{\varepsilon\pi} = -Dx \Rightarrow F_{\varepsilon\pi} = -k \cdot x \Rightarrow \underline{x = 0{,}1\sqrt{3}\ m}$

• Αρχή Διατήρησης της Ενέργειας Ταλάντωσης: Α.Δ.Ε.Τ.

$E = K + U \Rightarrow \frac{1}{2}kA^2 = \frac{1}{2}m_1 v^2 + \frac{1}{2}kx^2 \Rightarrow m_1 v^2 = k(A^2 - x^2) \Rightarrow$

$\Rightarrow v = \pm\sqrt{\frac{k}{m}} \cdot \sqrt{A^2 - x^2} \Rightarrow v = \pm\omega\sqrt{A^2 - x^2} \Rightarrow v = \pm 1 m/s$

Επειδή $x = 0{,}1\sqrt{3} m > 0$ και το σώμα απομακρύνεται από τη θέση ισορροπίας άρα $v > 0$ δηλαδή $\boxed{v = 1 m/s}$.

3.17 $L=0{,}5m$, $M=3kg$
F_1 (A)
F_2 (cm), $\vec{F_1} \parallel \vec{F_2}$, $t=2s$

A. $L = I_0 \omega_1$ ①

• Απο τη γραφική παράσταση $\omega = f(t)$ εχουμε $\underline{\omega_1 = 60\ rad/s}$

• Θεώρημα Steiner: $I_0 = I_{cm} + M\left(\frac{L}{2}\right)^2 \Rightarrow$

$\Rightarrow I_0 = \frac{1}{12}ML^2 + M\frac{L^2}{4} \Rightarrow I_0 = \frac{1}{3}ML^2 \Rightarrow \underline{I_0 = 0{,}25\ kgm^2}$

① $\Rightarrow \boxed{L = 15\ kgm^2/s}$

B. • $0 \le t \le 2s$: $a_{γων(1)} = \frac{\Delta\omega}{\Delta t} = \frac{60-0}{2-0}\ rad/s^2 \Rightarrow \underline{a_{γων(1)} = 30\ rad/s^2}$

• $\frac{dL}{dt} = \Sigma\tau = I_0 \cdot a_{γων(1)} \Rightarrow \boxed{\frac{dL}{dt} = 7{,}5\ kgm^2/s^2}$

• $\Sigma\tau = I_0 \cdot a_{γων(1)} \Rightarrow F_1 \cdot L = I_0 \cdot a_{γων(1)} \Rightarrow \boxed{F_1 = 15N}$

• $W_F = \tau_F \cdot \theta = F \cdot L \cdot \theta$
$\theta = \frac{1}{2} a_{γων(1)} t_1^2 \Rightarrow \theta = 60\ rad$
$\Rightarrow \boxed{W_F = 450J}$

Γ. • $2s \le t \le 6s$: $a_{γων(2)} = \frac{\Delta\omega}{\Delta t} = \frac{0-60}{6-2}\ rad/s^2$ ή $\underline{a_{γων(2)} = -15\ rad/s^2}$

• $\Sigma\tau = I_0 \cdot a_{γων(2)} \Rightarrow F_1 L - F_2 \frac{L}{2} = I_0 \cdot a_{γων(2)} \Rightarrow \boxed{F_2 = 45N}$

• $P_{F_2} = \tau_{F_2} \cdot \omega_2 = -F_2 \cdot \frac{L}{2}\omega_2$
$\omega_2 = \omega_0 - |a_{γων(2)}| \cdot \Delta t \Rightarrow \omega_2 = 60 - 15 \cdot (4-2)$ ή $\omega_2 = 30\ \frac{rad}{s}$
$\Rightarrow \boxed{P_{F_2} = 337{,}5W}$

Δ. • Επειδή η κλίση της γραφικής παράστασης απο το 2s-8s είναι σταθερή άρα $6s \le t \le 8s$: $a_{γων(3)} = a_{γων(2)} = -15\ rad/s^2$
όμως $a_{γων(3)} = \frac{\Delta\omega}{\Delta t} \Rightarrow -15 = \frac{\omega_T - 0}{8-6} \Rightarrow \underline{\omega_T = -30\ rad/s}$

• $\Delta L = L_8 - L_2 = I_0(\omega_T - \omega_2) = 0{,}25(-30-60)\ kgm^2/s$
ή $\boxed{\Delta L = -22{,}5\ kgm^2/s}$

Ε. 0-2s: $\theta_1 = \text{Εμβαδό} = \frac{1}{2}\cdot 2\cdot 60$ ή $\theta_1 = 60\,rad$

2s-6s: $\theta_2 = \text{Εμβαδό} = \frac{1}{2}(6-2)\cdot 60$ ή $\theta_2 = 120\,rad$

6s-8s: $\theta_3 = \text{Εμβαδό} = \frac{1}{2}(8-6)\cdot(-30)$ ή $\theta_3 = -30\,rad$

- $\theta_{ολ} = \theta_1 + \theta_2 + \theta_3 \Rightarrow \boxed{\theta_{ολ} = 150\,rad}$

- $N = \frac{|\theta_1| + |\theta_2| + |\theta_3|}{2\pi} \Rightarrow \boxed{N = \frac{105}{\pi}\text{ περιστροφές}}$

3.18 $L = 0{,}3\,m$, $M = 10\,kg$, $(B\Gamma) = \frac{L}{3}$

Α. Η δύναμη F έχει σταθερή ροπή

$W = \tau_F \cdot \theta = F\cdot(A\Gamma)\cdot\theta \xrightarrow[\theta = 2\pi\,rad]{(A\Gamma) = \ell - \ell/3} W = F\cdot\frac{2\ell}{3}\cdot 2\pi$

$\Rightarrow F = \frac{3\cdot W}{4\pi\ell} \Rightarrow \boxed{F = 10\,N}$

Β. • Θεώρημα Steiner: $I_\Gamma = I_{cm} + M(O\Gamma)^2 \xrightarrow{(O\Gamma) = \frac{L}{2} - \frac{L}{3} = \frac{L}{6}} I_\Gamma = \frac{1}{12}ML^2 + M\frac{L^2}{36}$

$\Rightarrow \underline{I_\Gamma = \frac{ML^2}{9} = 0{,}1\,kg\cdot m^2}$

- $\Sigma\tau = I_\Gamma \cdot a_{γων} \Rightarrow F\cdot(A\Gamma) = I_\Gamma\cdot a_{γων} \Rightarrow \boxed{a_{γων} = 20\,rad/s^2}$

Γ. • $\theta = N\cdot 2\pi \Rightarrow \underline{\theta = 40\,rad}$

- $\theta = \frac{1}{2} a_{γων} t_1^2 \Rightarrow \underline{t_1 = 2\,s}$
- $\omega = a_{γων} t_1 \Rightarrow \underline{\omega = 40\,rad/s}$
- $P_F = \tau_F\cdot\omega = F\cdot(A\Gamma)\cdot\omega = F\cdot\frac{2L}{3}\cdot\omega$ ή $\boxed{P_F = 80\,W}$

Δ. • $t = \frac{\omega_0}{|a'_{γων}|} \xrightarrow{\omega_0 = \omega = 40\,rad/s} \underline{|a'_{γων}| = 10\,rad/s^2}$

- Για να βρούμε την ελάχιστη τιμή της δύναμης F':

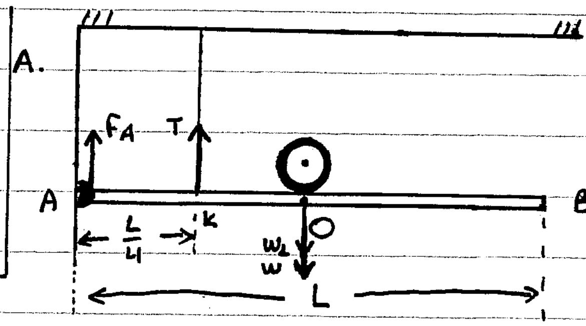
$\tau_{F'} = F'\cdot(A\Gamma)\cdot ημ\theta \Rightarrow F' = \frac{\tau_{F'}}{(A\Gamma)\cdot ημ\theta}$. Άρα η F' γίνεται ελάχιστη όταν $ημ\theta = 1 \Rightarrow \theta = 90°$, δηλαδή όταν ασκείται κάθετα στη ράβδο.

- $\Sigma\tau = I_\Gamma\cdot a'_{γων} \Rightarrow F\cdot(A\Gamma) - F'_{min}(B\Gamma) = I_\Gamma\cdot a'_{γων} \Rightarrow \boxed{F'_{min} = 30\,N}$

Ε. $\bar{P} = \frac{W}{t} = \frac{\Delta K}{t} = \frac{K_{τελ} - K_{αρχ}}{t} = \frac{-\frac{1}{2}I\omega^2}{t}$ ή $\boxed{\bar{P} = -20W}$

3.19 $L = 4m$
$M = 2\,kg$
$m = 2{,}5\,kg$, $r = 0{,}2m$
$(AK) = L/4$

Α.



• Το σύστημα ράβδος – σφαίρα ισορροπεί:

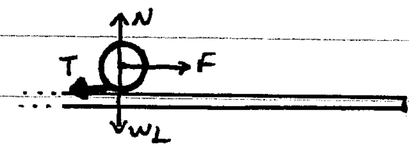
$\Sigma\tau_{(A)} = 0 \Rightarrow T\cdot(AK) - w\cdot(AO) - w_1\cdot(AO) = 0 \Rightarrow T\cdot\frac{L}{4} = Mg\frac{L}{2} + mg\frac{L}{2}$

$\Rightarrow \boxed{T = 90N}$

$\Sigma F = 0 \Rightarrow F_A + T = w + w_1 \Rightarrow F_A = Mg + mg - T \Rightarrow \underline{F_A = -45N}$

Άρα η F_A έχει μέτρο $\boxed{F_A = 45N}$ και φορά προς τα κάτω (αντίθετη από ότι σχεδιάσαμε).

Β.



Η σφαίρα κυλίεται χωρίς να ολισθαίνει

• $\Sigma\tau = I\cdot\alpha_{γων} \xrightarrow{\alpha_{γων} = \alpha_{cm}/r} T\cdot r = \frac{2}{5}mr^2\frac{\alpha_{cm}}{r} \Rightarrow$

$\Rightarrow T = \frac{2}{5}m\alpha_{cm}$ (1)

• $\Sigma F_x = m\alpha_{cm} \Rightarrow F - T = m\cdot\alpha_{cm} \xrightarrow{(1)} F - \frac{2}{5}m\alpha_{cm} = m\alpha_{cm} \Rightarrow$

$\Rightarrow F = \frac{7}{5}m\alpha_{cm} \Rightarrow \alpha_{cm} = \frac{5F}{7m} \Rightarrow \underline{\alpha_{cm} = 2\,m/s^2}$

• (1) $\Rightarrow \underline{T = 2N}$

• $\frac{dL}{dt} = \Sigma\tau = T\cdot r$ ή $\boxed{\frac{dL}{dt} = 0{,}4\,kgm^2/s}$

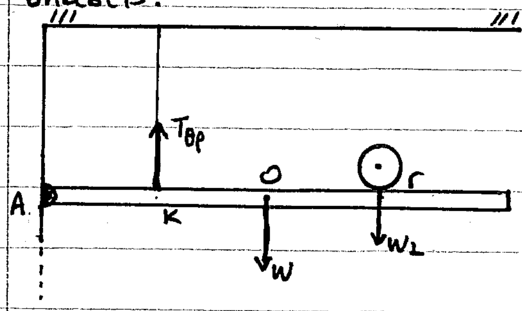
Γ. • Για να έχουμε κύλιση χωρίς ολίσθηση θα πρέπει η τριβή να είναι στατική: $T < \mu_s\cdot N \Rightarrow \mu_s > \frac{T}{N}$ (2)

• $\Sigma F_y = 0 \Rightarrow N = w_1 = mg \Rightarrow \underline{N = 25N}$

Άρα (2) $\Rightarrow \mu_s > 0{,}08$.

Αφού έχουμε $\mu_s = 0{,}20 > 0{,}08$ άρα δεν θα έχουμε ολίσθηση

Δ. Έστω ότι τη στιγμή t_1 η σφαίρα έχει φτάσει στο σημείο Γ της ράβδου και το νήμα είναι έτοιμο να σπάση.



$\cdot\ \Sigma\tau_{(A)} = 0 \Rightarrow T_{θρ}\cdot(AK) - W(AO) - W_1(Ar)$

$\Rightarrow (Ar) = \dfrac{T_{θρ}\cdot\frac{L}{4} - Mg\cdot\frac{L}{2}}{mg} \Rightarrow$

$\Rightarrow \underline{(Ar) = 3m}$

· Η σφαίρα διένυσε απόσταση $\Delta x = (Or) = (Ar) - (AO)$ ή $\underline{\Delta x = 1m}$

$\Delta x = \frac{1}{2}\alpha_{cm} t_1^2 \Rightarrow \underline{t_1 = 1s}$

$\omega = \alpha_{γων}\cdot t_1 \Rightarrow \omega = \frac{\alpha_{cm}}{r} t_1 \Rightarrow \underline{\omega = 10\ rad/s}$

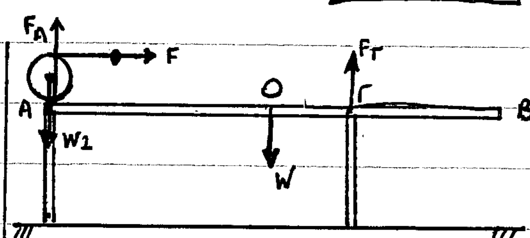
1). $L = I\cdot\omega \Rightarrow L = \frac{2}{5}mr^2\omega \Rightarrow \boxed{L = 0{,}4\ kg\,m^2/s}$

2) $\left.\begin{array}{l}\cdot\ P_F = F\cdot U_{cm} \\ U_{cm} = \omega\cdot r = 2\,m/s\end{array}\right\} \Rightarrow \boxed{P_F = 14W}$

$\cdot\ W_F = F\cdot x_{cm} \xrightarrow{x_{cm} = \Delta x = 1m} \boxed{W_F = 7J}$

3.20

$L = 3m$
$M = 4kg$
$(Or) = 0{,}5m$
$m = 4kg$, $R = 0{,}1m$



A. Η ράβδος ισορροπεί:

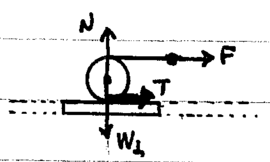
$\cdot\ \Sigma\tau_{(A)} = 0 \Rightarrow F_r\cdot(Ar) - W(AO)$

$= 0 \Rightarrow F_r = \dfrac{Mg\cdot\frac{L}{2}}{(\frac{L}{2} + Or)} \Rightarrow$

$\Rightarrow \boxed{F_r = 30N}$

$\cdot\ \Sigma F_y = 0 \Rightarrow F_A + F_r = W + W_1 \Rightarrow \boxed{F_A = 50N}$

B.



$\cdot\ \Sigma\tau = I\alpha_{γων} \Rightarrow F\cdot R - TR = \frac{mR^2}{2}\alpha_{γων} \xrightarrow{\alpha_{γων} = \alpha_{cm}/R}$

$F - T = \frac{mR}{2}\cdot\frac{\alpha_{cm}}{R} \Rightarrow T = F - \frac{m\alpha_{cm}}{2}$ (1)

$\cdot\ \Sigma F_x = m\alpha_{cm} \Rightarrow F + T = m\alpha_{cm} \xrightarrow{(1)} F + F - \frac{m\alpha_{cm}}{2} = m\alpha_{cm} \Rightarrow 2F = \frac{3}{2}m\alpha_{cm}$

$\Rightarrow \alpha_{cm} = \frac{4F}{3m} \Rightarrow \boxed{\alpha_{cm} = 5m/s^2 \left(= \frac{dU_{cm}}{dt}\right)}$

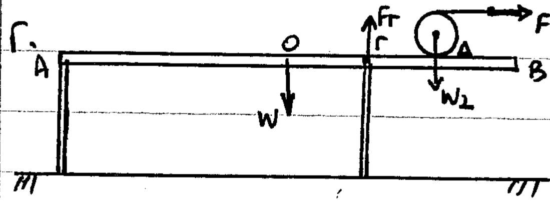
$\cdot$ (1) $\Rightarrow T = 5N$

$\cdot\ \Sigma F_y = 0 \Rightarrow N - W_1 = 0 \Rightarrow N = mg \Rightarrow \underline{N = 40N}$,

· Για να έχουμε κύλιση χωρίς ολίσθηση θα πρέπει $T \le \mu_s\cdot N$

$\Rightarrow \mu_s \ge \frac{T}{N} \Rightarrow \mu_s \ge 0{,}125$

Επειδή $\mu_s = 0{,}2 > 0{,}125$ άρα ο κύλινδρος κυλίεται χωρίς να ολισθαίνει



Έστω ότι τη στιγμή t1 που πάει να αρχίσει η ανατροπή της ράβδου, ο κύλινδρος βρίσκεται στη θέση Δ, τότε ισχύει $F_A=0$

- $\Sigma\tau_{(\Gamma)}=0 \Rightarrow W\cdot(O\Gamma)-W_1\cdot(\Gamma\Delta)=0 \Rightarrow (\Gamma\Delta)=\frac{W}{W_1}\cdot(O\Gamma) \Rightarrow (\Gamma\Delta)=0,5m$
- Άρα ο κύλινδρος μέχρι τη στιγμή t1 θα έχει διανύσει απόσταση $\Delta x=(A\Delta)=2,5m$

$$\Delta x=\frac{1}{2}a_{cm}t_1^2 \Rightarrow t_1=\sqrt{\frac{2\cdot\Delta x}{a_{cm}}} \Rightarrow \boxed{t_1=1s}$$

- Το νήμα που θα έχει ξετυλιχτεί θα είναι:

$$\Delta\theta=\frac{1}{2}a_{γων}t_1^2 \xrightarrow{a_{γων}=a_{cm}/R=50\,rad/s^2} \Delta\theta=25\,rad$$

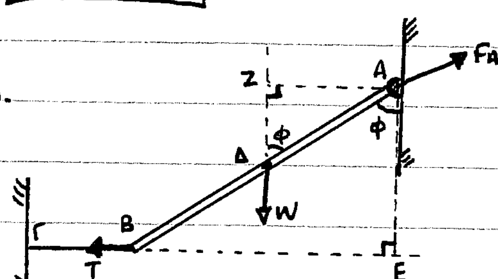
$$\Delta\theta=\frac{s}{R} \xrightarrow{s=\ell} \ell=\Delta\theta\cdot R \Rightarrow \boxed{\ell=2,5m}$$

Δ. $\left.\begin{matrix} L=I\omega \\ \omega=a_{γων}t_1=50\,rad/s \end{matrix}\right\} \Rightarrow L=\frac{1}{2}mR^2\omega \Rightarrow \boxed{L=1\,kgm^2/s}$

- $P_F=F\cdot\upsilon_{cm}+\tau_F\cdot\omega \Rightarrow P_F=F\cdot\upsilon_{cm}+F\cdot R\omega \xrightarrow{\upsilon_{cm}=\omega R} P_F=2F\upsilon_{cm} \Rightarrow$
$\xrightarrow{\upsilon_{cm}=\omega R=5m/s} \boxed{P_F=150\,W}$

3.21 $M=6kg$, $(AB)=1m$, $ημφ=0,6$, $συνφ=0,8$

A.



Η ράβδος ισορροπεί:

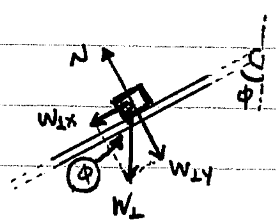
- $\Sigma\tau_{(A)}=0 \Rightarrow W\cdot(AZ)-T(AE)=0$
$\Rightarrow T=Mg\frac{(AZ)}{(AE)}$ (1)

- $\overset{\Delta}{AZ\Delta}$: $ημφ=\frac{(AZ)}{(A\Delta)} \Rightarrow (AZ)=\frac{\ell}{2}\cdot ημφ=0,3$

$\overset{\Delta}{AEB}$: $συνφ=\frac{(AE)}{(AB)} \Rightarrow (AE)=\ell\cdot συνφ=0,8$

Άρα (1) $\Rightarrow T=6\cdot 10\,\frac{0,3}{0,8}\,N \Rightarrow \boxed{T=22,5\,N}$

B.

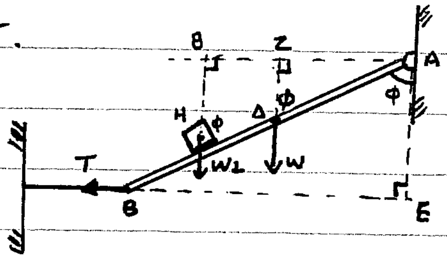


$\Sigma F_x=m\cdot a_{cm} \Rightarrow W_{1x}=m\,a_{cm} \Rightarrow \boxed{a_{cm}=8\frac{m}{s^2}=\left(\frac{d\upsilon_{cm}}{dt}\right)}$

$W_{1x}=W_1 συνφ=mg\,συνφ=16N$

$W_{1y}=W_1 ημφ=mg\,ημφ=12N$

Γ.



Έστω ότι τη στιγμή που είναι έτοιμο να σπάσει το νήμα το σώμα έχει φτάσει στο σημείο Η της ράβδου

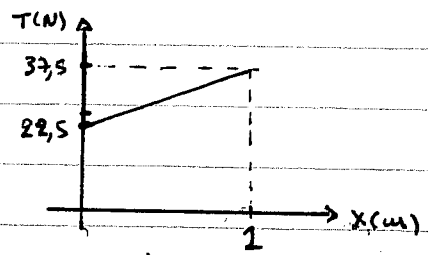
$\Sigma\tau_{(A)}=0 \Rightarrow w\cdot(AZ)+w_1\cdot(A\Theta) - T_{op}\cdot(AE)=0$ (2)

· $\overset{\triangle}{A\Theta H}$: $\eta\mu\phi=\frac{(A\Theta)}{(AH)} \Rightarrow \underline{(A\Theta)=(AH)\cdot\eta\mu\phi=0{,}6\cdot x}$, ($(AH)=x$: η απόσταση που διένυσε το σώμα)

$(2) \Rightarrow 60\cdot 0{,}3+20\cdot 0{,}6\cdot x=37{,}5\cdot 0{,}8 \Rightarrow 12x=30-18 \Rightarrow \boxed{x=1m}$

Αφού $x=1m=(AB)$, το νήμα θα σπάσει όταν το σώμα βρεθεί στο άκρο Β. της ράβδου.

Δ. $(2) \Rightarrow 18+12x=T\cdot 0{,}8 \Rightarrow \underline{T=22{,}5+15\cdot x}$ (3) (S.I.)

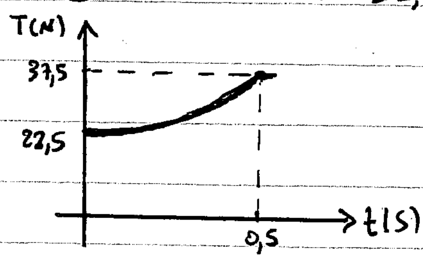


$(3) \xRightarrow{x=0} T=22{,}5N$

$(3) \xRightarrow{x=1m} T=37{,}5N$

Το νήμα θα σπάσει τη στιγμή: $x=\frac{1}{2}a_{cm}t^2 \Rightarrow t=\sqrt{\frac{2x}{a_{cm}}} \xRightarrow{x=1m} \underline{t=0{,}5s}$

$(3) \xRightarrow{x=\frac{1}{2}a_{cm}t^2=4t^2} T=22{,}5+60t^2$ (4)



$(4) \xRightarrow{t=0} T=22{,}5N$

$(4) \xRightarrow{t=0{,}5s} T=37{,}5N$

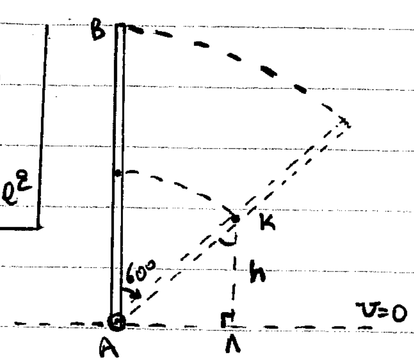
Ε. · $\left.\begin{array}{l} K=\frac{1}{2}mU_{cm}^2 \\ U_{cm}=a_{cm}t=4m/s \end{array}\right\} \Rightarrow \boxed{K=16J}$

· $\left.\begin{array}{l} \frac{dK}{dt}=\Sigma F_x\cdot U_{cm}=W_x\cdot U_{cm} \\ W_x=mg\,\sigma\upsilon\nu\phi=16N \end{array}\right\} \Rightarrow \boxed{\frac{dK}{dt}=64\ J/s}$

3.22 $\ell=30cm$

$M=1kg$

$I_{cm}=\frac{1}{12}M\ell^2$



· Η κίνηση της ράβδου είναι μη ομαλά επιταχυνόμενη και εφαρμόζουμε ΑΔΜΕ.

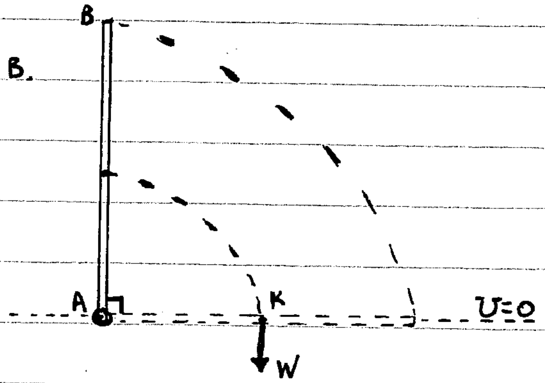
$E_{MHX}^{\alpha\rho\chi}=E_{MHX}^{\tau\varepsilon\lambda} \Rightarrow K_{\alpha\rho\chi}+U_{\alpha\rho\chi}=K_{\tau\varepsilon\lambda}+U_{\tau\varepsilon\lambda} \Rightarrow$

$\Rightarrow Mg\frac{\ell}{2}=K+Mgh$ (1)

· $\overset{\triangle}{A\Lambda K}$: $\sigma\upsilon\nu 60^\circ=\frac{(\Lambda K)}{(AK)} \Rightarrow h=\frac{\ell}{2}\cdot\sigma\upsilon\nu 60^\circ \Rightarrow \underline{h=\frac{\ell}{4}}$

① $\Rightarrow K = Mg\frac{\ell}{2} - Mg\frac{\ell}{4} \Rightarrow \boxed{K = 0{,}75\ J}$

Β.



• Α.Δ.Μ.Ε.: $E_{MHX}^{αρχ} = E_{MHX}^{τελ} \Rightarrow K_{αρχ} + U_{αρχ} = K_{τελ} + U_{τελ}$

$\Rightarrow Mg\frac{\ell}{2} = \frac{1}{2} I_A \cdot \omega^2$ ②

• Θεώρημα Steiner: $I_A = I_{cm} + M\frac{\ell^2}{4} \Rightarrow$

$\Rightarrow I_A = \frac{1}{12}M\ell^2 + \frac{1}{4}M\ell^2 \Rightarrow I_A = \frac{M\ell^2}{3}$

② $\Rightarrow Mg\frac{\ell}{2} = \frac{1}{2}\cdot\frac{M\ell^2}{3}\cdot\omega^2 \Rightarrow \omega = \sqrt{\frac{3g}{\ell}} \Rightarrow \underline{\omega = 10\ rad/s}$

• $L = I_A\cdot\omega \Rightarrow L = \frac{1}{3}M\ell^2\omega \Rightarrow \boxed{L = 0{,}3\ kg\,m^2/s}$

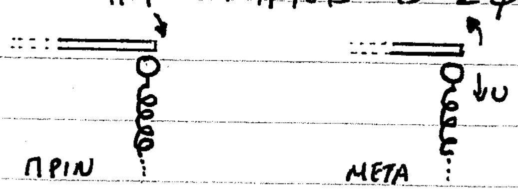
• $\upsilon_K = \omega\cdot\frac{\ell}{2} \Rightarrow \boxed{\upsilon_K = 1{,}5\ m/s}$

Γ. • $\Sigma\tau = I_A \cdot a_{γων} \Rightarrow W\cdot\frac{\ell}{2} = \frac{M\ell^2}{3}\cdot a_{γων} \Rightarrow Mg\frac{\ell}{2} = \frac{M\ell^2}{3}\cdot a_{γων} \Rightarrow a_{γων} = \frac{3g}{2\ell} \Rightarrow$

$\Rightarrow \boxed{a_{γων} = 50\ rad/s^2}$

Δ. • κρούση ράβδου - σώματος μάζας m.

Αρχή Διατήρησης της Στροφορμής: $\vec{L}_{αρχ} = \vec{L}_{τελ}$



$I_A\cdot\omega = -I_A\cdot\frac{\omega}{2} + m\upsilon\ell \Rightarrow$

$\Rightarrow \frac{3}{2}\frac{M\ell^2}{3}\cdot\omega = m\upsilon\ell \Rightarrow \upsilon = \frac{M\ell\omega}{m\cdot 2} \Rightarrow$

$\Rightarrow \boxed{\upsilon = 1{,}5\ m/s}$

• α.α.Τ. του σώματος m

• Η κρούση έγινε στη θέση ισορροπίας η οποία δεν αλλάζει μετά την κρούση άρα: $\upsilon = \upsilon_{max} = \omega A$

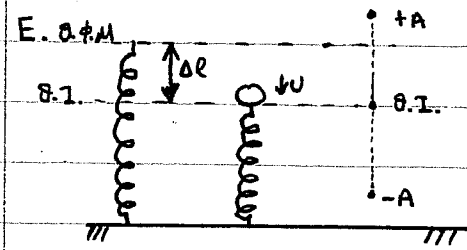
$\omega = \sqrt{\frac{k}{m}} = 10\ rad/s$

$\Rightarrow \underline{A = 0{,}15\ m}$

• $y = A\eta\mu(\omega t + \varphi_0) \xrightarrow[y=0,\ \upsilon<0]{t_0=0} 0 = A\eta\mu\varphi_0 \xrightarrow{A\neq 0} \eta\mu\varphi_0 = 0 = \eta\mu 0 \Rightarrow$

$\begin{cases} \varphi_0 = 2k\pi + 0 \\ \varphi_0 = 2k\pi + \pi - 0 \end{cases} \xRightarrow{k=0} \begin{cases} \varphi = 0,\ \upsilon > 0 \\ \boxed{\varphi_0 = \pi},\ \upsilon < 0 \end{cases}$

Άρα $y = A\eta\mu(\omega t + \varphi_0) \Rightarrow \boxed{y = 0{,}15\,\eta\mu(10t + \pi)}$, (S.I.)



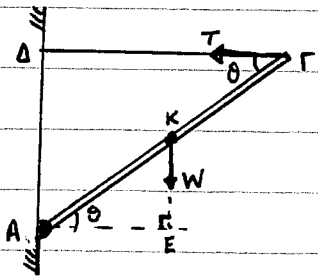
• Θ.Ι.: $\Sigma F = 0 \Rightarrow F_{\varepsilon\lambda} - w = 0 \Rightarrow k \cdot \Delta\ell = mg \Rightarrow$

$\Rightarrow \Delta\ell = \frac{mg}{k} \Rightarrow \Delta\ell = 0{,}1m$

• $x = +A$: $F_{\varepsilon\lambda} = k(A - \Delta\ell) \Rightarrow \boxed{F_{\varepsilon\lambda} = 5N}$

• $x = -A$: $F'_{\varepsilon\lambda} = k(A + \Delta\ell) \Rightarrow \boxed{F'_{\varepsilon\lambda} = 25N}$

3.23 $\ell = 1{,}5m$, $m = 2kg$
$\eta\mu\theta = 0{,}8$, $\sigma\upsilon\nu\theta = 0{,}6$
$I_{cm} = \frac{1}{12} m\ell^2$, $g = 10 \frac{m}{s^2}$



A. Η ράβδος ισορροπεί:

$\Sigma\tau_{(A)} = 0 \Rightarrow T \cdot (A\Delta) - w \cdot (AE) = 0$ (1)

• $\overset{\triangle}{A\Delta\Gamma}$: $\eta\mu\theta = \frac{(A\Delta)}{(A\Gamma)} \Rightarrow (A\Delta) = \ell\eta\mu\theta$

• $\overset{\triangle}{AEK}$: $\sigma\upsilon\nu\theta = \frac{(AE)}{(AK)} \Rightarrow (AE) = \frac{\ell}{2}\sigma\upsilon\nu\theta$

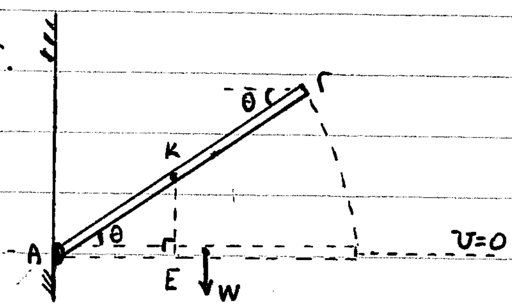
(1) $\Rightarrow T = mg\frac{(AE)}{(A\Delta)} \Rightarrow T = mg\frac{\frac{\ell}{2}\sigma\upsilon\nu\theta}{\ell\eta\mu\theta} \Rightarrow \boxed{T = 7{,}5N}$

B. • $\Sigma\tau = I_A \cdot \alpha_{\gamma\omega\nu} \Rightarrow w \cdot (AE) = I_A \cdot \alpha_{\gamma\omega\nu} \Rightarrow \alpha_{\gamma\omega\nu} = \frac{mg \cdot \frac{\ell}{2}\sigma\upsilon\nu\phi}{I_A}$ (2).

• Θεώρημα Steiner: $I_A = I_{cm} + m\frac{\ell^2}{4} \Rightarrow I_A = \frac{1}{12}m\ell^2 + m\frac{\ell^2}{4} \Rightarrow I_A = \frac{1}{3}m\ell^2$

(2) $\Rightarrow \alpha_{\gamma\omega\nu} = \frac{mg\frac{\ell}{2}\sigma\upsilon\nu\phi}{\frac{1}{3}m\ell^2} \Rightarrow \alpha_{\gamma\omega\nu} = \frac{3g\sigma\upsilon\nu\phi}{2\ell} \Rightarrow \boxed{\alpha_{\gamma\omega\nu} = 6 rad/s^2}$

Γ.



Α.Δ.Μ.Ε.: $E_{MHX}^{\alpha\rho\chi} = E_{MHX}^{\tau\varepsilon\lambda} \Rightarrow K_{\alpha\rho\chi} + U_{\alpha\rho\chi} = K_{\tau\varepsilon\lambda} + U_{\tau\varepsilon\lambda}$

$\Rightarrow mg(KE) = \frac{1}{2}I_A\omega^2$ (3)

• $\overset{\triangle}{AEK}$: $\eta\mu\theta = \frac{(KE)}{(AK)} \Rightarrow (KE) = \frac{\ell}{2} \cdot \eta\mu\theta$

(3) $\Rightarrow mg\frac{\ell}{2}\eta\mu\theta = \frac{1}{2}\frac{1}{3}m\ell^2\omega^2 \Rightarrow$

$\Rightarrow \omega = \sqrt{\frac{3g\eta\mu\theta}{\ell}} \Rightarrow \boxed{\omega = 4 rad/s}$

• $L = I_A \cdot \omega \Rightarrow L = \frac{1}{3}m\ell^2\omega \Rightarrow \boxed{L = 6 kgm^2/s}$

Δ. Το βάρος είναι συντηρητική δύναμη άρα:

$W_w = -\Delta U \Rightarrow W_w = U_{\alpha\rho\chi} - U_{\tau\varepsilon\lambda} \Rightarrow W_w = mg(KE) \Rightarrow W_w = mg\frac{\ell}{2}\eta\mu\theta$

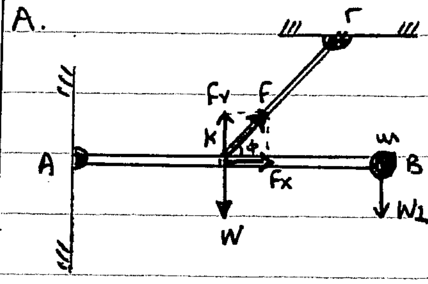
$\Rightarrow \boxed{W_w = 12J}$

• $P_w = \tau_w \cdot \omega \Rightarrow P_w = w \cdot \frac{\ell}{2}\omega \Rightarrow P_w = mg\frac{\ell}{2}\omega \Rightarrow \boxed{P_w = 60W}$

3.24

$L = 1m$, $M = 6kg$
$m = 2kg$
$\phi = 30^\circ$
$(K\Gamma)$: αβαρής ράβδος

A.



Οι δυνάμεις που ασκούνται στην αβαρή ράβδο (ΚΓ) από την οροφή και τη ράβδο (ΑΒ) έχουν τη διεύθυνση της ράβδου (ΚΓ) ώστε να μπορούν να δώσουν $\Sigma\tau = 0$, γιατί σε διαφορετική περίπτωση θα είχαμε $\Sigma\tau \neq 0$ αφού θα αποτελούσαν ζεύγος δυνάμεων. Άρα η δύναμη που ασκεί η ράβδος (ΚΓ) στη ράβδο (ΑΒ) θα είναι κατά μήκος της αβαρούς ράβδου.

1) $\Sigma\tau_{(A)} = 0 \Rightarrow F_y \cdot \frac{L}{2} - w\frac{L}{2} - w_1 \cdot L = 0 \Rightarrow F\eta\mu\phi\frac{L}{2} = Mg\frac{L}{2} + mgL$

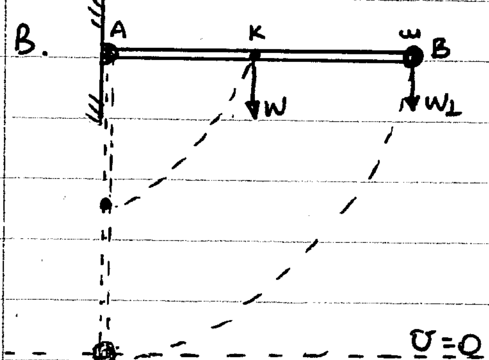
$\Rightarrow \boxed{F = 200N}$

2) $\cdot I_A = I_A^{\rho\acute{\alpha}\beta\delta o\upsilon} + I_A^{m}$ (1)

$\cdot$ Θεώρημα Steiner: $I_A = I_{cm} + M\frac{L^2}{4} \Rightarrow I_A = \frac{1}{12}ML^2 + \frac{ML^2}{4} \Rightarrow I_A = \frac{ML^2}{3}$

(1) $\Rightarrow I_A = \frac{ML^2}{3} + mL^2 \Rightarrow \boxed{I_A = 4\,kg \cdot m^2}$

B.



1) $\cdot \Sigma\tau_{(A)} = I_A \cdot \alpha_{\gamma\omega\nu} \Rightarrow w \cdot \frac{L}{2} + w_1 L = I_A \cdot \alpha_{\gamma\omega\nu}$

$\Rightarrow I_A \cdot \alpha_{\gamma\omega\nu} = Mg\frac{L}{2} + mgL \Rightarrow \boxed{\alpha_{\gamma\omega\nu} = 12{,}5\,\frac{rad}{s^2}}$

2) $\cdot$ <u>Α.Δ.Μ.Ε.</u>: $E_{\mu\eta\chi}^{\alpha\rho\chi} = E_{\mu\eta\chi}^{\tau\varepsilon\lambda} \Rightarrow K_{\alpha\rho\chi} + U_{\alpha\rho\chi} = K_{\tau\varepsilon\lambda} + U_{\tau\varepsilon\lambda}$

$\Rightarrow MgL + mgL = \frac{1}{2}I_A\omega^2 + Mg\frac{L}{2} \Rightarrow$

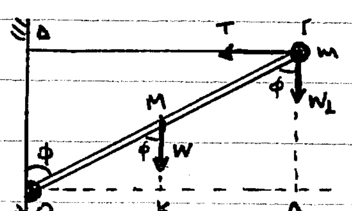
$\Rightarrow \underline{\omega = 5\,rad/s}$

Άρα $\upsilon_B = \omega L \Rightarrow \boxed{\upsilon_B = 5\,m/s}$

3.25

$L = 1{,}5m$, $W = 24N$
$m = 0{,}8kg$, $\phi = 60^\circ$

A.



$\cdot$ Το σύστημα ράβδος - σώμα m ισορροπεί

$\Sigma\tau_{(O)} = 0 \Rightarrow T \cdot (O\Delta) - W \cdot (OK) - W_1(O\Lambda) = 0$ (1)

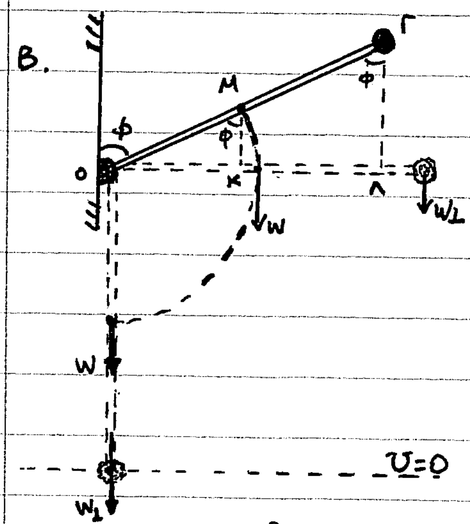
$\cdot\ \overset{\Delta}{O\Delta\Gamma}$: $\sigma\upsilon\nu\phi = \frac{(O\Delta)}{(O\Gamma)} \Rightarrow \underline{(O\Delta) = L\sigma\upsilon\nu\phi = \frac{L}{2}}$

$\cdot\ \overset{\Delta}{OKM}$: $\eta\mu\phi = \frac{(OK)}{(OM)} \Rightarrow \underline{(OK) = \frac{L}{2}\eta\mu\phi = \frac{L\sqrt{3}}{4}}$

$\cdot\ \overset{\Delta}{O\Lambda\Gamma}$: $\eta\mu\phi = \frac{(O\Lambda)}{(O\Gamma)} \Rightarrow \underline{(O\Lambda) = L\eta\mu\phi = \frac{L\sqrt{3}}{2}}$

① $\Rightarrow T\cdot\frac{L}{2} = w\,\frac{L\sqrt{\ }}{4} + mg\,\frac{L\sqrt{\ }}{2} \Rightarrow \boxed{T = 20\sqrt{3}\,N}$

B.



$\frac{dL}{dt} = \Sigma\tau = w\cdot\frac{L}{2} + mg\cdot L$ ή $\boxed{\frac{dL}{dt} = 30\ kg\,m^2/s^2}$

Γ. 1) ΑΔΜΕ: $E_{MHX}^{αρχ} = E_{MHX}^{τελ} \Rightarrow K_{αρχ} + U_{αρχ} = K_{τελ} + U_{τελ} \Rightarrow$

$\Rightarrow Mg\,[L + (MK)] + mg\,[L + (ΓΛ)] = \frac{1}{2} I_O \omega^2 + Mg\frac{L}{2}$ ②

• $O\hat{K}M$: $\sigma\upsilon\nu\phi = \frac{(MK)}{(OM)} \Rightarrow (MK) = \frac{L}{2}\sigma\upsilon\nu\phi = \frac{L}{4}$

• $O\hat{Λ}Γ$: $\sigma\upsilon\nu\phi = \frac{(ΓΛ)}{(OΓ)} \Rightarrow (ΓΛ) = L\sigma\upsilon\nu\phi = \frac{L}{2}$

• Θεώρημα Steiner: $I_O^{ράβδου} = I_{cm} + M\frac{L^2}{4} \Rightarrow$

$I_O^{ράβδου} = \frac{1}{12}ML^2 + M\frac{L^2}{4} \Rightarrow I_O^{ράβδου} = \frac{ML^2}{3}$

Άρα $I_O = I_O^{ράβδου} + I_O^{m} \Rightarrow I_O = \frac{ML^2}{3} + mL^2 \xrightarrow{W = Mg \Rightarrow M = 2,4kg} \underline{I_O = 3,6\,kg\,m^2}$

$② \overset{S.I.}{\Rightarrow} 24\left(1,5 + \frac{1,5}{4}\right) + 8\left(1,5 + \frac{1,5}{2}\right) = \frac{1}{2}\cdot 3,6\cdot\omega^2 + 24\cdot\frac{1,5}{2} \Rightarrow$

$\Rightarrow 45 + 18 = 1,8\omega^2 + 18 \Rightarrow \omega^2 = 25 \Rightarrow \boxed{\omega = 5\ rad/s}$

2) $\cdot\ \Sigma\tau = I_O\cdot\alpha_{γων} \Rightarrow 0 = I_O\cdot\alpha_{γων} \Rightarrow \boxed{\alpha_{γων} = 0}$

$\cdot\ U = \omega L \Rightarrow \boxed{U = 7,5\ m/s}$

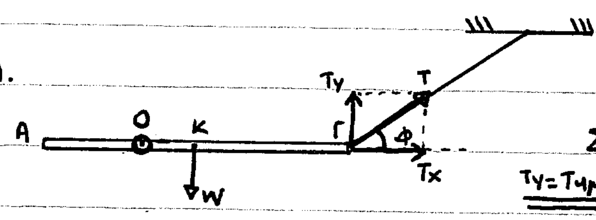
3)

$F_K = m\frac{U^2}{L} \xrightarrow{F_K = \Sigma F} F - mg = m\frac{U^2}{L} \Rightarrow F = mg + m\frac{U^2}{L} \Rightarrow \boxed{F = 38N}$

3.26

$L = 1m$, $m = 10kg$
$(OA) = L/3$
$\phi = 30^\circ$
$I_{cm} = \frac{1}{12}mL^2$

A.



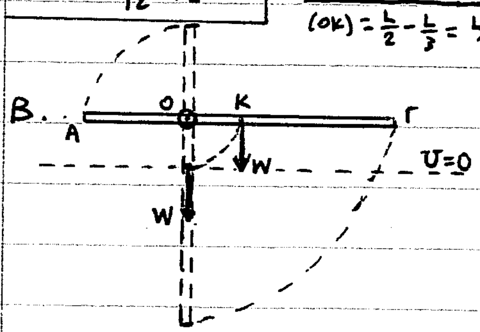
Η ράβδος ισορροπεί:

$\Sigma\tau_{(O)} = 0 \Rightarrow -W\cdot(OK) + T_y\cdot(OΓ) = 0$

$\xrightarrow{T_y = T\eta\mu\phi} T\eta\mu\phi\cdot(OΓ) = mg\,(OK) \Rightarrow$

$\xrightarrow[(OK) = \frac{L}{2} - \frac{L}{3} = L/6]{(OΓ) = L - \frac{L}{3} = 2L/3} T\cdot\frac{1}{2}\cdot\frac{2L}{3} = mg\,\frac{L}{6} \Rightarrow T = \frac{mg}{2} \Rightarrow \boxed{T = 50N}$

B.



1) $\cdot\ \Sigma\tau = I_O\cdot\alpha_{γων} \Rightarrow W\cdot(OK) = I_O\cdot\alpha_{γων}$ ①

$\cdot$ Θεώρημα Steiner: $I_O = I_{cm} + m(OK)^2 \Rightarrow$

$\Rightarrow I_O = \frac{1}{12}mL^2 + m\frac{L^2}{36} \Rightarrow \underline{I_O = \frac{mL^2}{9}}$

$① \Rightarrow mg\cdot\frac{L}{6} = m\frac{L^2}{9}\alpha_{γων} \Rightarrow \boxed{\alpha_{γων} = 15\ rad/s^2}$

2) · Λόγω ύπαρξης τριβών θα εφαρμόσω την αρχή διατήρησης της ενέργειας. <u>Α.Δ.Ε</u> : $mg\frac{L}{6} = \frac{1}{2}I_O\omega^2 + Q \Rightarrow \boxed{Q = \frac{70}{9}J \ (=E_{απωλειών})}$

· <u>2ος τρόπος</u>: ΘΜΚΕ

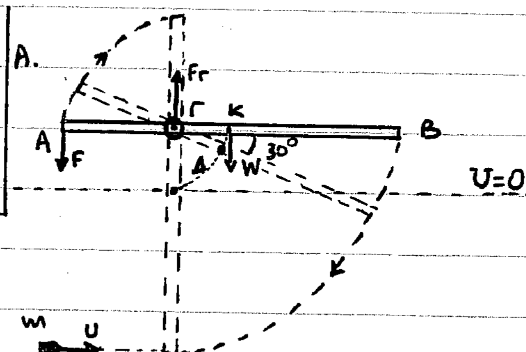
$$\Sigma W = \Delta K \Rightarrow W_w + W_T = K_{τελ} - \cancel{K_{αρχ}}^{0} \Rightarrow mg\frac{L}{6} + W_T = \frac{1}{2}I_O\omega^2 \Rightarrow$$

$$\Rightarrow W_T = -\frac{70}{9}J$$ ή $E_{απωλειών} = Q = |W_T| = \frac{70}{9}J$.

3) $\boxed{\frac{dL}{dt} = \Sigma\tau = 0}$

3.27 $\ell = 60cm$, $M = 1kg$, $(AΓ) = \ell/3$, $I_{cm} = \frac{1}{12}M\ell^2$

A.



· Η ροπή της δύναμης F που ασκούμε στο άκρο Α της ράβδου είναι: $\tau_F = F\cdot(AΓ)\cdot ημθ \Rightarrow F = \frac{\tau_F}{(AΓ)\cdot ημθ}$

Η δύναμη F ελαχιστοποιείται όταν ημθ=1 ή θ=90°, δηλαδή αν ασκηθεί κάθετα στη ράβδο.

· $\Sigma\tau_{(Γ)} = 0 \Rightarrow F\cdot(AΓ) - w(ΓK) = 0 \Rightarrow F = Mg\frac{(ΓK)}{(AΓ)} \xrightarrow[(AΓ) = L/3]{(ΓK) = \frac{L}{2} - \frac{L}{3} = \frac{L}{6}} \boxed{F = 5N}$

(με $\vec{F} \uparrow\uparrow \vec{w}$)

· $\Sigma F = 0 \Rightarrow F_Γ - F - W = 0 \Rightarrow F_Γ = F + Mg \Rightarrow \boxed{F_Γ = 15N}$

B. · <u>Α.Δ.Μ.Ε.</u>: $E_{ΜΗΧ}^{αρχ} = E_{ΜΗΧ}^{τελ} \Rightarrow K_{αρχ} + U_{αρχ} = K_{τελ} + U_{τελ} \Rightarrow$

$$Mg(ΓK) = \frac{1}{2}I_Γ\omega^2 + Mg[(ΓK) - (ΓK)\cdot ημ30] \Rightarrow Mg(ΓK)ημ30 = \frac{1}{2}I_Γ\omega^2 \quad (1)$$

· Θεώρημα Steiner: $I_Γ = I_{cm} + M(ΓK)^2 \Rightarrow I_Γ = \frac{1}{12}ML^2 + \frac{ML^2}{36} \Rightarrow I_Γ = \frac{ML^2}{9}$

$$(1) \Rightarrow Mg\cdot\frac{L}{6}\cdot\frac{1}{2} = \frac{1}{2}\cdot\frac{ML^2}{9}\omega^2 \Rightarrow \boxed{\omega = 5\ rad/s}$$

$$υ_A = \omega\cdot(AΓ) \Rightarrow υ_A = \omega\frac{L}{3} \Rightarrow \boxed{υ_A = 1\ m/s}$$

Γ. · <u>Α.Δ.Μ.Ε.</u>: $E_{ΜΗΧ}^{αρχ} = E_{ΜΗΧ}^{τελ} \Rightarrow K_{αρχ} + U_{αρχ} = K_{τελ} + U_{τελ} \Rightarrow Mg(ΓK) = \frac{1}{2}I_Γ\omega'^2 \Rightarrow$

$$\Rightarrow Mg\frac{L}{6} = \frac{1}{2}\frac{ML^2}{9}\omega'^2 \Rightarrow \underline{\omega' = 5\sqrt{2}\ rad/s}$$

· $L = I_Γ\cdot\omega' \Rightarrow \boxed{L = 0{,}2\sqrt{2}\ kgm^2/s}$

Δ. Επειδή για το σύστημα ράβδου - βλήματος μάζας m έχουμε ότι $\Sigma\tau_{εξ}=0$ άρα μπορούμε να εφαρμόσουμε την αρχή διατήρησης της στροφορμής.

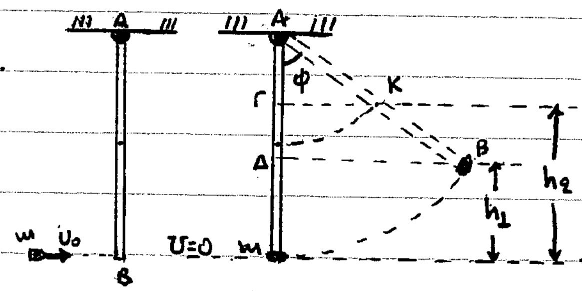
<u>Α.Δ.Στρ.:</u> $\vec{L}_{αρχ}=\vec{L}_{τελ}$

$$mv(ΓΒ) - I_Γ\cdot\omega' = 0 \Rightarrow v = \frac{I_Γ\,\omega'}{m(ΓΒ)} \xrightarrow{(ΓΒ)=L-\frac{L}{3}=\frac{2L}{3}}$$

$$\Rightarrow v = \frac{\frac{ML^2}{9}\cdot\omega'}{m\cdot\frac{2L}{3}} \Rightarrow \boxed{v=1\,m/s}$$

3.28 $\ell=0{,}45m$, M
$m=M$, $\upsilon_0=3m/s$

Α.



Εφαρμόζουμε την αρχή διατήρησης της στροφορμής, αφού για το σύστημα ράβδου - βλήματος έχουμε $\Sigma\tau_{εξ}=0$. <u>Α.Δ.Στρ.:</u> $\vec{L}_{αρχ}=\vec{L}_{τελ}$

$$m\upsilon_0\ell = I_A\cdot\omega \quad (1)$$

· $I_A = I_A^{ράβδου}+I_A^{m}$

Θεώρημα Steiner: $I_A^{ράβδου} = I_{cm}+M\frac{\ell^2}{4} = \frac{1}{12}M\ell^2+M\frac{\ell^2}{4}=\frac13 M\ell^2$

$$\Rightarrow I_A = \frac13 M\ell^2 + m\ell^2 \xrightarrow{M=m} \underline{I_A=\frac43 m\ell^2}$$

$$(1) \Rightarrow m\upsilon_0\ell = \frac43 m\ell^2\omega \Rightarrow \omega = \frac{3\upsilon_0}{4\ell} \Rightarrow \boxed{\omega=5\,rad/s}$$

Β. <u>Α.Δ.Μ.Ε.:</u> $E_{ΜΗΧ}^{αρχ}=E_{ΜΗΧ}^{τελ} \Rightarrow K_{αρχ}+U_{αρχ}=K_{τελ}+U_{τελ}\Rightarrow$

$$\frac12 I_A\omega^2 + Mg\frac{\ell}{2} = mgh_1+Mgh_2 \xrightarrow{M=m} \frac23\ell^2\omega^2+g\frac{\ell}{2}=gh_1+gh_2 \quad (2)$$

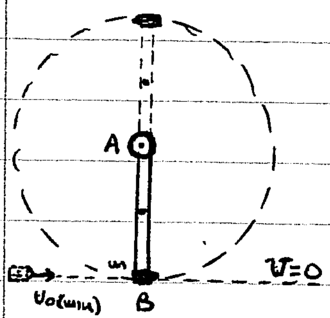
· $\hat{ΑΔΒ}$: $συνφ = \frac{(ΑΔ)}{(ΑΒ)} \Rightarrow (ΑΔ)=\ell\, συνφ$ άρα $h_1=\ell-(ΑΔ)\Rightarrow \underline{h_1=\ell-\ell\, συνφ}$

· $\hat{ΑΓΚ}$: $συνφ=\frac{(ΑΓ)}{(ΑΚ)}\Rightarrow (ΑΓ)=\frac{\ell}{2}συνφ$ άρα $h_2=\ell-(ΑΓ)\Rightarrow \underline{h_2=\ell-\frac{\ell}{2}συνφ}$

$$(2) \Rightarrow \frac23\ell^2\omega^2+g\frac{\ell}{2}=g(\ell-\ell\, συνφ)+g\left(\ell-\frac{\ell}{2}συνφ\right) \xrightarrow{S.I.} \frac23\cdot 0{,}45\cdot 25+5=10-10συνφ+$$

$$+10-5συνφ \Rightarrow 12{,}5-20=-15συνφ \Rightarrow συνφ=\frac{7{,}5}{15}\Rightarrow συνφ=\frac12 \text{ ή } \boxed{φ=60^\circ}$$

Γ. Για να καταφέρει το σύστημα ράβδος - βλήμα να κάνει οριακή ανακύκλωση, θα πρέπει να φτάσει στο ανώτερο σημείο της τροχιάς της με μηδενική ταχύτητα ($\omega=0$).



• Α.Δ.Μ.Ε.: $E_{MHX}^{αρχ} = E_{MHX}^{τελ} \Rightarrow K_{αρχ}+U_{αρχ} = K_{τελ}+U_{τελ} \Rightarrow$

$$\xrightarrow{K_{τελ}=0} \frac{1}{2} I_A\omega_{min}^2 + Mg\frac{L}{2} = Mg\frac{3L}{2} + mg2L \xrightarrow{M=m}$$

$$\frac{2}{3} \ell^2\omega_{min}^2 + g\frac{\ell}{2} = g\frac{3\ell}{2} + g2\ell \Rightarrow$$

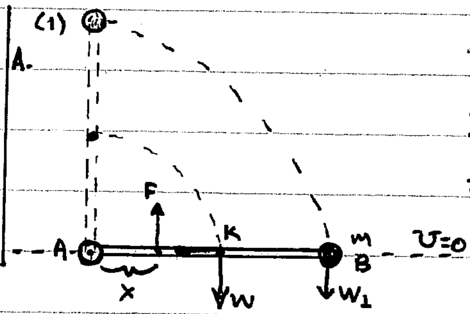
$$\frac{2}{3} \ell\omega_{min}^2 + \frac{g}{2} = \frac{3g}{2}+g\cdot 2 \Rightarrow \omega_{min}=10\ rad/s$$

• Α.Δ. Στροφορμής: $\vec{L}_{αρχ} = \vec{L}_{τελ}$

$$m\upsilon_{0(min)}\ell = I_A\cdot\omega \Rightarrow \boxed{\upsilon_{0(min)}=6\ m/s}$$

Δ. $\Sigma\tau = \frac{\Delta L}{\Delta t} \Rightarrow F\cdot\ell = \frac{I_A^{ράβδου}\omega_{min}}{\Delta t} \Rightarrow F\cdot\ell = \frac{\frac{M\ell^2}{3}\cdot\omega_{min}}{\Delta t} \Rightarrow \boxed{F=450N}$

3.29 $L=1m$, $M=3kg$, $m=1kg$, $F=100N$, $I_{cm}=\frac{1}{12}ML^2$



Το σύστημα ισορροπεί:

$\Sigma\tau_{(A)}=0 \Rightarrow F\cdot x - w\frac{L}{2} - w_1 L = 0$

$\Rightarrow x = \frac{Mg\frac{L}{2}+mgL}{F} \Rightarrow \boxed{x=0{,}25m}$

Β. Επειδή στο σύστημα ασκείται η εξωτερική δύναμη $F=\frac{400}{\pi}N$ θα εφαρμόσουμε θεώρημα μεταβολής κινητικής ενέργειας (ή θεώρημα έργου ενέργειας).

$$\Sigma W = \Delta K \Rightarrow W_F + W_w + W_{w_1} = K_{τελ} - K_{αρχ} \quad (1)$$

• $W_F = \tau_F\Delta\theta = F\cdot x\cdot\Delta\theta = F\cdot x\cdot\frac{\pi}{2}$ ή $W_F = 50J$

• $W_w = -\Delta U = U_{αρχ}-U_{τελ} = -Mg\frac{L}{2}$ ή $W_w = -15J$

• $W_{w_1} = -\Delta U = U_{αρχ}-U_{τελ} = -mgL$ ή $W_{w_1} = -10J$

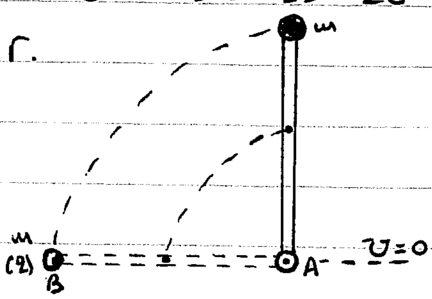
• $I_A = I_A^{ράβδου} + I_A^{m}$

Θεώρημα Steiner: $I_A^{ράβδου} = I_{cm} + M\frac{L^2}{4} = \frac{1}{12}ML^2 + M\frac{L^2}{4} = \frac{1}{3}ML^2$ $\Rightarrow$

$\Rightarrow I_A = \frac{1}{3}ML^2 + mL^2 \Rightarrow I_A = 2\ kg\cdot m^2$

(1) $\Rightarrow 50-15-10 = \frac{1}{2}\cdot 2\omega^2 \Rightarrow \boxed{\omega_1 = 5\ rad/s}$

Γ.

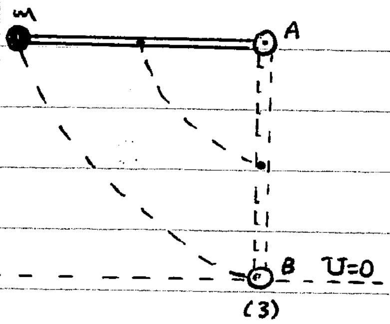


• Α.Δ.Μ.Ε.: $E_{MHX}^{αρχ} = E_{MHX}^{τελ} \Rightarrow K_{αρχ}+U_{αρχ} = K_{τελ}+U_{τελ}$

$\Rightarrow \frac{1}{2}I_A\omega_1^2 + Mg\frac{L}{2} + mgL = \frac{1}{2}I_A\omega_2^2 \Rightarrow \omega = 5\sqrt{2}\ \frac{rad}{s}$

• $L = I_A\cdot\omega \Rightarrow \boxed{L = 10\sqrt{2}\ kg\,m^2/s}$

Δ.



• ΘΜΚΕ: $\Sigma W = \Delta K \Rightarrow W_{F'} + W_w + W_{w_1} = K_{\tau\varepsilon\lambda} - K_{\alpha\rho\chi}$ (2)

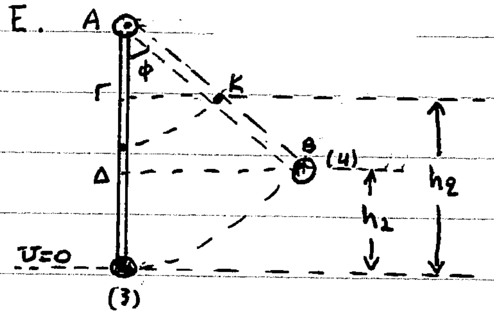
$W_w = -\Delta U = U_{\alpha\rho\chi} - U_{\tau\varepsilon\lambda} = Mgl - Mg\frac{l}{2} = Mg\frac{l}{2} = 15J$

$W_{w_1} = -\Delta U = U_{\alpha\rho\chi} - U_{\tau\varepsilon\lambda} = mgL = 10J$

(2) $\Rightarrow W_{F'} + 15 + 10 = \frac{1}{2}2\,25 - \frac{1}{2}2\cdot 50 \Rightarrow$

$\Rightarrow \boxed{W_{F'} = -50J}$

Ε.



• Α.Δ.Μ.Ε.: $E_{MHX}^{\alpha\rho\chi} = E_{MHX}^{\tau\varepsilon\lambda} \Rightarrow K_{\alpha\rho\chi} + U_{\alpha\rho\chi} =$

$K_{\tau\varepsilon\lambda} + U_{\tau\varepsilon\lambda} \Rightarrow \frac{1}{2}I_A\omega_3^2 + Mg\frac{l}{2} = Mgh_2 + mgh_1$ (3)

• $A\hat{\Delta}B$: $\sigma\upsilon\nu\phi = \frac{(A\Delta)}{(AB)} \Rightarrow (A\Delta) = l\sigma\upsilon\nu\phi$, άρα $h_1 = l - l\sigma\upsilon\nu\phi$

• $A\hat{\Gamma}K$: $\sigma\upsilon\nu\phi = \frac{(A\Gamma)}{(AK)} \Rightarrow (A\Gamma) = \frac{l}{2}\sigma\upsilon\nu\phi$, άρα $h_2 = l - \frac{l}{2}\sigma\upsilon\nu\phi$

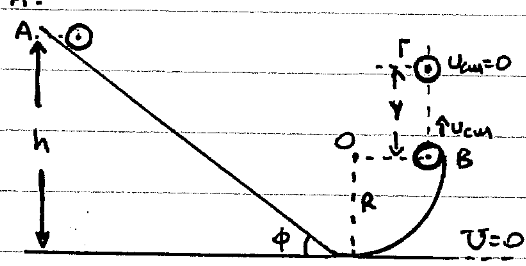
(3) $\overset{S.I.}{\Rightarrow} \frac{1}{2}\cdot 2\cdot 25 + 15 = 30(1-\sigma\upsilon\nu\phi) + 10(1-\frac{1}{2}\sigma\upsilon\nu\phi) \Rightarrow$

$\Rightarrow 4 = 3 - 3\sigma\upsilon\nu\phi + 1 - \frac{1}{2}\sigma\upsilon\nu\phi \Rightarrow 3{,}5\sigma\upsilon\nu\phi = 0 \Rightarrow \sigma\upsilon\nu\phi = 0$ ή $\boxed{\phi = 90^\circ}$

3.30

$m = 1kg$, $r = 0{,}02m$
$I_{cm} = \frac{2}{5}mr^2$
$h = 9m$
$R = 2m$, $r \ll R$

Α.



• Θεώρημα Steiner:

$I_B = I_{cm} + mr^2 \Rightarrow I_B = \frac{2}{5}mr^2 + mr^2 \Rightarrow$

$\Rightarrow I_B = \frac{7}{5}mr^2 \Rightarrow \boxed{I_B = 5{,}6\cdot 10^{-4}\,kgm^2}$

Β. • Α.Δ.Μ.Ε.: $E_{MHX}^{\alpha\rho\chi} = E_{MHX}^{\tau\varepsilon\lambda} \Rightarrow K_{\alpha\rho\chi} + U_{\alpha\rho\chi} = K_{\tau\varepsilon\lambda} + U_{\tau\varepsilon\lambda} \Rightarrow$

$\Rightarrow mgh = mgR + \frac{1}{2}mu_{cm}^2 + \frac{1}{2}I_{cm}\omega^2 \Rightarrow mg(h-R) = \frac{1}{2}mu_{cm}^2 + \frac{1}{5}mr^2\omega^2 \Rightarrow$

$\xRightarrow{u_{cm}=\omega r} mg(h-R) = \frac{7}{10}mu_{cm}^2 \Rightarrow u_{cm} = \sqrt{\frac{10}{7}g(h-R)} \Rightarrow \boxed{u_{cm} = 10m/s}$

• $\frac{K_{\mu\varepsilon\tau}}{K_{\sigma\tau\rho}} = \frac{\frac{1}{2}mu_{cm}^2}{\frac{1}{2}\cdot\frac{2}{5}mr^2\omega^2} = \frac{u_{cm}^2}{\frac{2}{5}u_{cm}^2}$ ή $\boxed{\frac{K_{\mu\varepsilon\tau}}{K_{\sigma\tau\rho}} = \frac{5}{2}}$

Γ. • $L = I_{cm}\omega \xRightarrow{\omega = \frac{u_{cm}}{r}} L = \frac{2}{5}mr^2\frac{u_{cm}}{r} \Rightarrow \boxed{L = 0{,}08\,kgm^2/s}$

• $\frac{K}{K_B} = \frac{\frac{1}{2}mu_{cm}^2 + \frac{1}{2}I_{cm}\omega^2}{\frac{1}{2}I_B\cdot\omega^2} = \frac{mu_{cm}^2 + \frac{2}{5}mr^2\frac{u_{cm}^2}{r^2}}{\frac{7}{5}mr^2\frac{u_{cm}^2}{r^2}} = \frac{mu_{cm}^2(1+\frac{2}{5})}{\frac{7}{5}mu_{cm}^2}$

ή $\boxed{\frac{K}{K_B} = 1}$ Παρατήρηση: ως προς άξονα περιστροφής που διέρχεται από το B η σφαίρα εκτελεί μόνο στροφική κίνηση.

Δ. • Α.Δ.Μ.Ε. (από το Β στο Γ): $E_{μηχ}^{αρχ} = E_{μηχ}^{τελ} \Rightarrow K_{αρχ} + U_{αρχ} = K_{τελ} + U_{τελ} \Rightarrow$

$$\Rightarrow \frac{1}{2} m v_{cm}^2 + \frac{1}{2} I_{cm}\omega^2 + mgR = \frac{1}{2} I_{cm}\omega^2 + mg(R+y) \Rightarrow$$

$$\Rightarrow \frac{1}{2} m v_{cm}^2 = mgy \Rightarrow y = \frac{v_{cm}^2}{2g} \Rightarrow \boxed{y = 5m}$$

• Η σφαίρα εκτελεί σύνθετη κίνηση (από το Β στο Γ) από μεταφορική σκοπιά: ομαλά επιβραδυνόμενη, ενώ από στροφική σκοπιά: ομαλή κυκλική ($\Sigma\tau = 0$), άρα:

$$v = v_0 - |a_{cm}|t \xRightarrow{v=0} t = \frac{v_0}{|a_{cm}|} \quad (1)$$

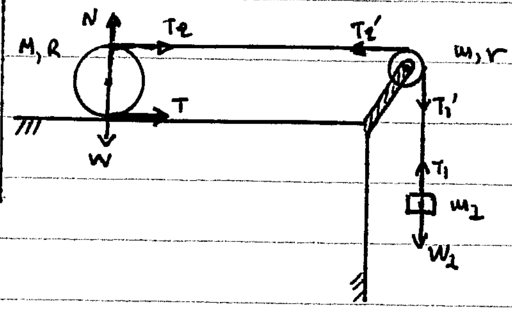
$$\Sigma F = m a_{cm} \Rightarrow -W = m a_{cm} \Rightarrow -mg = m a_{cm} \Rightarrow a_{cm} = -g = -10 m/s^2$$

$$(1) \Rightarrow \boxed{t = 1s}$$

3.1 $M = 4kg$, $R = 0,1m$

$m = 1kg$, $r = 5cm$

$m_1 = 2kg$



A. • Σώμα m_1: $\Sigma F = m_1 a_{cm} \Rightarrow$

$\Rightarrow w_1 - T_1 = m_1 a_{cm} \Rightarrow$

$\Rightarrow \underline{T_1 = m_1 g - m_1 a_{cm}}$ (1)

• Τροχαλία: $\Sigma\tau = I_{cm} \cdot \alpha_{\gamma\omega\nu} \xrightarrow{\alpha_{\gamma\omega\nu} = a_{cm}/r} T_1' r - T_2' r = \frac{1}{2} m r^2 \frac{a_{cm}}{r} \Rightarrow$

$\xrightarrow[T_2 = T_2']{T_1 = T_1'} m_1 g - m_1 a_{cm} - T_2 = \frac{1}{2} m a_{cm} \Rightarrow \underline{T_2 = m_1 g - m_1 a_{cm} - \frac{1}{2} m a_{cm}}$ (2)

• Τροχός: $\Sigma F_x = M \cdot a'_{cm} \Rightarrow T_2 + T = M a'_{cm} \xrightarrow[(2)]{a_{cm} = 2 \cdot a'_{cm}} m_1 g - m_1 a_{cm} - \frac{1}{2} m a_{cm} +$

$+ T = M \frac{a_{cm}}{2} \Rightarrow \underline{T = M \frac{a_{cm}}{2} - m_1 g + m_1 a_{cm} + \frac{1}{2} m a_{cm}}$ (3)

$\Sigma\tau = I_{cm} \cdot \alpha_{\gamma\omega\nu} \xrightarrow{\alpha'_{\gamma\omega\nu} = a'_{cm}/R} T_2 R - T R = \frac{1}{2} M R^2 \frac{a'_{cm}}{R} \xrightarrow{(2)(3)}$

$m_1 g - m_1 a_{cm} - \frac{1}{2} m a_{cm} - M \frac{a_{cm}}{2} + m_1 g - m_1 a_{cm} - \frac{1}{2} m a_{cm} = \frac{1}{4} M a_{cm} \Rightarrow$

$a_{cm} \left(\frac{M}{4} + 2m_1 + \frac{M}{2} + m \right) = 2 m_1 g \Rightarrow \boxed{a_{cm} = 5 m/s^2}$

B. $\left(\frac{dL}{dt}\right)_{\tau\rho o\chi\alpha\lambda\iota\alpha\varsigma} = \Sigma\tau = I \cdot \alpha_{\gamma\omega\nu} = \frac{1}{2} m r^2 \frac{a_{cm}}{r} = \frac{1}{2} m r a_{cm}$ ή $\boxed{\left(\frac{dL}{dt}\right)_{\tau\rho o\chi\alpha\lambda.} = 0,125 \frac{kg m^2}{s^2}}$

Γ. $\frac{K_\Delta}{K_{\tau\rho}} = \frac{\frac{1}{2} M v_{cm}'^2 + \frac{1}{2} I \omega'^2}{\frac{1}{2} I \omega^2} = \frac{M v_{cm}'^2 + \frac{1}{2} M R^2 \omega'^2}{\frac{1}{2} m r^2 \omega^2} = \frac{M v_{cm}'^2 + \frac{1}{2} M v_{cm}'^2}{\frac{1}{2} m v_{cm}^2} \xrightarrow{v_{cm} = 2 v'_{cm}}$

$\frac{K_\Delta}{K_{\tau\rho}} = \frac{\frac{3}{2} M v_{cm}'^2}{\frac{1}{2} m 4 v_{cm}'^2} \Rightarrow \frac{K_\Delta}{K_{\tau\rho}} = \frac{3M}{4m}$ ή $\boxed{\frac{K_\Delta}{K_{\tau\rho}} = 3}$

Δ. • (2) $\Rightarrow T_2 = (20 - 10 - 2,5) N \Rightarrow \underline{T_2 = 7,5 N}$

• $P_{T_2} = T_2 \cdot v'_{cm} + \tau_{T_2} \cdot \omega' \Rightarrow P_{T_2} = T_2 v'_{cm} + T_2 \cdot R \cdot \omega' \xrightarrow{v'_{cm} = \omega' R} P_{T_2} = 2 \cdot T_2 \cdot v'_{cm}$

$\Rightarrow v'_{cm} = \frac{7,5}{2 \cdot 7,5} m/s \Rightarrow \underline{v'_{cm} = 0,5 m/s}$

• $v'_{cm} = a'_{cm} t_1 \xrightarrow{a'_{cm} = \frac{a_{cm}}{2}} t_1 = \frac{0,5}{2,5} s \Rightarrow \underline{t_1 = 0,2 s}$

• $\theta = \frac{1}{2} \alpha'_{\gamma\omega\nu} t_1^2 \Rightarrow \theta = \frac{1}{2} \frac{a'_{cm}}{R} t_1^2 \Rightarrow \underline{\theta = 0,5 rad}$

• $\theta = \frac{s}{R} \xrightarrow{s = \ell} \ell = \theta \cdot R \Rightarrow \boxed{\ell = 0,05 m}$

E. $n\% = \frac{K_1}{W_{w_1}} \cdot 100\%$ (4)

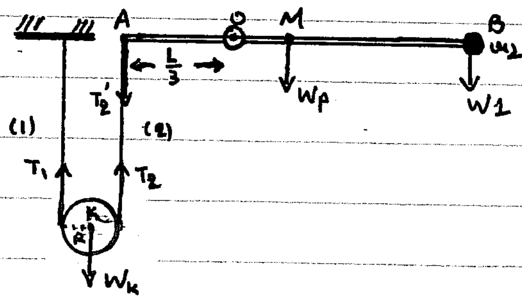
• $K_1 = \frac{1}{2} m_1 v_{cm}^2$, $v_{cm} = a_{cm} t_1 = 1 m/s$ $\Big\} \Rightarrow \underline{K_1 = 1J}$

• $W_{w_1} = w_1 \cdot y = m_1 g y$, $y = \frac{1}{2} a_{cm} t_1^2 = 0,1m$ $\Big\} \Rightarrow \underline{W_{w_1} = 2J}$

• (4) $\Rightarrow \Pi\% = \frac{1}{2} 100\% \Rightarrow \boxed{\Pi\% = 50\%}$

3.9. $m = 3kg$, $R = 0,1m$, $M = 1kg$, $L = 1,5m$, $(AO) = \frac{L}{3}$, m_1

A.



Το σύστημα ράβδος - σώμα m_1 ισορροπεί, άρα:

$\Sigma\tau_{(O)} = 0 \Rightarrow T_2' \cdot \frac{L}{3} - w_\rho \cdot \left(\frac{L}{2} - \frac{L}{3}\right) - w_1 \cdot \left(L - \frac{L}{3}\right) = 0 \Rightarrow T_2' \frac{L}{3} = Mg\frac{L}{6} + m_1 g \frac{2L}{3}$

$\Rightarrow T_2' = \frac{Mg}{2} + 2m_1 g$ (1)

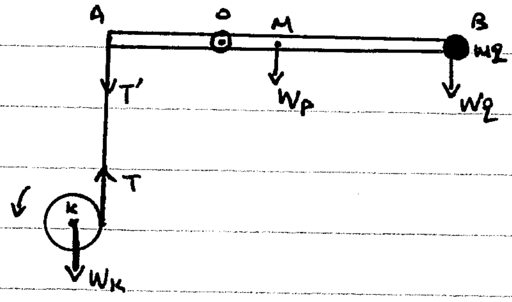
Ο κύλινδρος ισορροπεί:

• $\Sigma\tau_{(K)} = 0 \Rightarrow T_2 R - T_1 R = 0 \Rightarrow \underline{T_1 = T_2}$ (2)

• $\Sigma F = 0 \Rightarrow T_1 + T_2 - w_k = 0 \xRightarrow{(2)} 2T_1 = mg \Rightarrow \boxed{T_1 = 15N (= T_2)}$

(1) $\xRightarrow{T_2 = T_2'} \underline{m_1 = 0,5 kg}$

B.



1) Ο κύλινδρος εκτελεί σύνθετη κίνηση:

• $\Sigma\tau = I_{cm} \alpha_{\gamma\omega\nu} \xrightarrow{\alpha_{\gamma\omega\nu} = a_{cm}/R} T \cdot R = \frac{1}{2} m R^2 \frac{a_{cm}}{R} \Rightarrow$

$\Rightarrow T = \frac{1}{2} m a_{cm}$ (3)

• $\Sigma F = m a_{cm} \Rightarrow w_k - T = m a_{cm} \xRightarrow{(3)}$

$mg - \frac{1}{2} m a_{cm} = m a_{cm} \Rightarrow \underline{a_{cm} = \frac{2}{3} g = \frac{20}{3} \frac{m}{s^2}}$

• (3) $\Rightarrow \underline{T = 10N}$

• $\frac{dL}{dt} = \Sigma\tau = TR$ ή $\boxed{\frac{dL}{dt} = 1 kg m^2/s^2}$

2) Το σύστημα ράβδος - σώμα m_2 ισορροπεί οριζόντιο, άρα:

$\Sigma\tau_{(0)}=0 \Rightarrow T'\frac{L}{3} - w_\rho\left(\frac{L}{2}-\frac{L}{3}\right) - w_2\left(L-\frac{L}{3}\right) = 0 \xrightarrow{T'=T=10N}$

$\Rightarrow T'\frac{L}{3} - Mg\frac{L}{6} = m_2 g\frac{2L}{3} \Rightarrow m_2 = 0{,}25\ Kg$

• $\frac{m_1}{m_2} = \frac{0{,}5}{0{,}25} \Rightarrow \boxed{\frac{m_1}{m_2}=2}$

Γ. • $\Delta\theta = N\cdot 2\pi \Rightarrow \Delta\theta = \frac{6}{\pi}\cdot 2\pi\ rad \Rightarrow \Delta\theta = 12\ rad$

• $\Delta\theta = \frac{1}{2} a_{\gamma\omega\nu} t_1^2 \xrightarrow{a_{\gamma\omega\nu}=a_{cm}/R} t_1=\sqrt{\frac{2\cdot\Delta\theta}{a_{\gamma\omega\nu}}} \Rightarrow t_1=0{,}6s$

• $\Delta\theta=\frac{\Delta s}{R} \xrightarrow{\Delta s=\Delta \ell} \Delta\ell=\Delta\theta\cdot R \Rightarrow \boxed{\Delta\ell=1{,}2m}$

• $U_{cm}=a_{cm}t \Rightarrow \boxed{U_{cm}=4m/s}$

Δ. • Από 0-0,6s : ο κύλινδρος εκτελεί σύνθετη κίνηση
- Μεταφορικά: επιταχυνόμενη ($\upsilon_0=0$), $a_{cm}=\frac{20}{3}m/s^2$
- Στροφικά: επιταχυνόμενη ($\omega_0=0$), $a_{\gamma\omega\nu}=\frac{200}{3}\frac{rad}{s^2}$

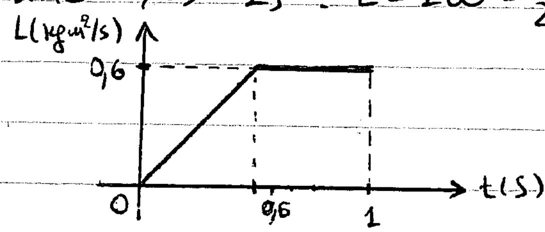
• Από 0,6s-1s : ο κύλινδρος εκτελεί σύνθετη κίνηση
- Μεταφορικά: επιταχυνόμενη ($\upsilon_0=4m/s$), $a_{cm}=g=10\frac{m}{s^2}$
- Στροφικά: ομαλή κυκλική κίνηση ($\omega=\frac{\upsilon_0}{R}=40\frac{rad}{s}$=σταθ.)

Άρα από 0-0,6s : $L=I\omega=\frac{1}{2}mR^2 a_{\gamma\omega\nu}t$ ή $L=t$ (4) (S.I.)

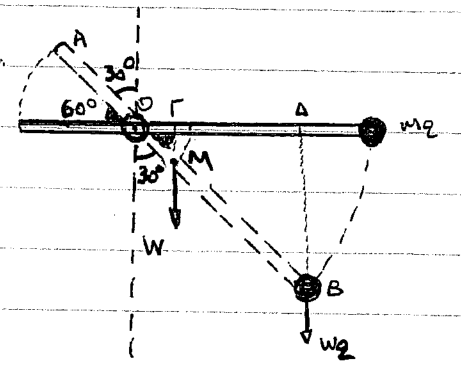
(4) $\xrightarrow{t=0}$ $L=0$

(4) $\xrightarrow{t=0,6s}$ $L=0{,}6\ kgm^2/s$

από 0,6s-1s : $L=I\omega=\frac{1}{2}mR^2\omega \Rightarrow L=0{,}6\ kgm^2/s$ =σταθ.



Ε.



• $I_O = I_O^{\rho\acute{\alpha}\beta\delta o\upsilon} + I_O^{m_2}$ (5)

• $I_O^{\rho\acute{\alpha}\beta\delta o\upsilon} = I_{cm} + M\cdot(OM)^2 = \frac{1}{12}ML^2 + M\left(\frac{L}{2}-\frac{L}{3}\right)^2 \Rightarrow$

$I_O^{\rho\acute{\alpha}\beta\delta o\upsilon} = \frac{1}{12}ML^2+\frac{ML^2}{36} \Rightarrow I_O^{\rho\acute{\alpha}\beta\delta o\upsilon}=\frac{ML^2}{9}$

(1) $\Rightarrow I_O=\frac{ML^2}{9}+m_2\left(L-\frac{L}{3}\right)^2 \Rightarrow$

$\Rightarrow I_O=\frac{ML^2}{9}+m_2\frac{4L^2}{9} \Rightarrow I_O=0{,}5\ kgm^2$

• $\Sigma\tau = I_o \cdot a_{\gamma\omega\nu} \Rightarrow w\cdot(O\Gamma) + w_2\cdot(O\Delta) = I_o \cdot a_{\gamma\omega\nu}$ (6)

• $O\hat{\Gamma}M$: $\sigma\upsilon\nu 60^\circ = \frac{(O\Gamma)}{(OM)} \Rightarrow (O\Gamma) = (OM)\cdot\sigma\upsilon\nu 60^\circ \Rightarrow (O\Gamma) = \frac{L}{6}\cdot\frac{1}{2} = \frac{L}{12}$

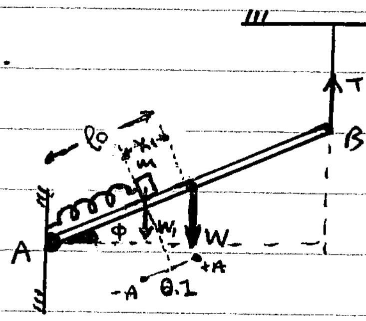
• $O\hat{\Delta}B$: $\sigma\upsilon\nu 60^\circ = \frac{(O\Delta)}{(OB)} \Rightarrow (O\Delta) = \frac{2L}{3}\cdot\sigma\upsilon\nu 60^\circ \Rightarrow (O\Delta) = \frac{L}{3}$

(6) $\Rightarrow a_{\gamma\omega\nu} = \dfrac{Mg\cdot\frac{L}{12} + m_2 g\cdot\frac{L}{3}}{I_o} \Rightarrow \boxed{a_{\gamma\omega\nu} = 5\ rad/s^2}$

3.3

$M = 4kg$
$(AB) = \ell = 2m$
$T_{max} = 25N$
$\eta\mu\varphi = 0{,}8,\ \sigma\upsilon\nu\varphi = 0{,}6$
$m = 1kg,\ k = 16N/m$
$\ell_0 = 1m$

A.



• Θ.Ι.(m): $\Sigma F_x = 0 \Rightarrow F_{\varepsilon\lambda} - w_{1x} = 0 \Rightarrow kx_1 = mg\eta\mu\varphi$
$\Rightarrow x_1 = 0{,}5m$

• Το σύστημα ράβδου – σώματος m ισορροπεί:
$\Sigma\tau_{(A)} = 0 \Rightarrow T\cdot\ell\cdot\sigma\upsilon\nu\varphi - w\frac{\ell}{2}\sigma\upsilon\nu\varphi - w_1\left(\frac{\ell}{2} - x_1\right)\cdot\sigma\upsilon\nu\varphi = 0 \xRightarrow{S.I.} T\cdot 1{,}2 = 24 + 3 \Rightarrow \boxed{T = 22{,}5N}$

B. • Η ταχύτητα υ_0 που έχει το σώμα m στη θέση ισορροπίας αντιστοιχεί στη μέγιστη ταχύτητα ταλάντωσής του:

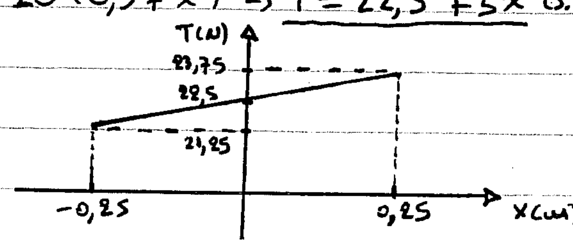
$\upsilon_0 = \upsilon_{max} = \omega A$
$\omega = \sqrt{\frac{k}{m}} \Rightarrow \omega = 4\ rad/s$
$\Rightarrow A = 0{,}25m$

$t_0 = 0,\ x = 0$ (Θ.Ι.), $\upsilon > 0$: $\varphi_0 = 0$

Άρα $\boxed{x = 0{,}25\eta\mu 4t}$ (S.I.) (1)

Γ. • $\Sigma\tau_{(A)} = 0 \Rightarrow T\cdot\ell\cdot\sigma\upsilon\nu\varphi - w\cdot\frac{\ell}{2}\sigma\upsilon\nu\varphi - w_1\cdot\left(\frac{\ell}{2} - x_1 + x\right)\sigma\upsilon\nu\varphi = 0 \Rightarrow$
$\xRightarrow{S.I.} 2T = 40 + 10(0{,}5 + x) \Rightarrow T = 22{,}5 + 5x$ (S.I.) (2)

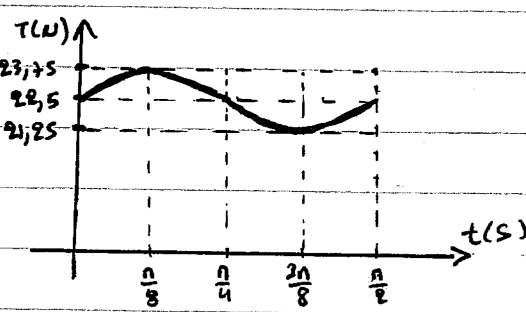
▪ (2) $\xRightarrow{x=A}$ $T = 23{,}75N$
▪ (2) $\xRightarrow{x=0}$ $T = 22{,}5N$
▪ (2) $\xRightarrow{x=-A}$ $T = 21{,}25N$



• (2) $\xRightarrow{(1)}$ $T = 22{,}5 + 1{,}25\eta\mu 4t$ (S.I.) (3)

▪ (3) $\xRightarrow{t=0}$ $T = 22{,}5N$

- (3) $\xRightarrow{t=T/4}$ $T = 23{,}75N$
- (3) $\xRightarrow{t=T/2}$ $T = 22{,}5N$
- (3) $\xRightarrow{t=3T/4}$ $T = 21{,}25N$
- (3) $\xRightarrow{t=T}$ $T = 22{,}5N$



Δ. - Έστω ότι το σώμα m έχει απομακρυνθεί κατά x από τη θέση ισορροπίας του τη στιγμή που η τάση του νήματος είναι ίση με $T_{max} = 25N$

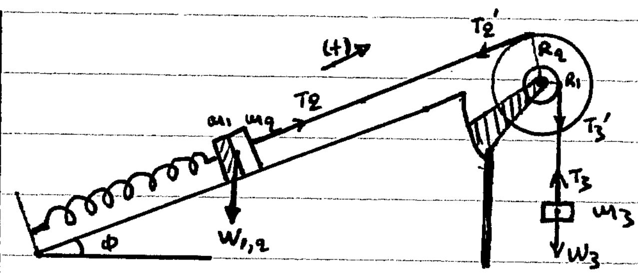
$\Sigma\tau_{(A)} = 0 \Rightarrow T_{max} \cdot \ell \cdot \sigma\upsilon\nu\varphi - w\frac{\ell}{2}\sigma\upsilon\nu\varphi - w_1\left(\frac{\ell}{2} - x_1 + x\right) = 0$

$\xRightarrow{S.I.} 50 - 40 - 5 - 10x = 0 \Rightarrow \underline{x = 0{,}5m}$

- Για να μη σπάσει το νήμα θα πρέπει το πλάτος ταλάντωσης να είναι $A \leq 0{,}5m \xRightarrow{U_{max} = \omega A} \frac{U_{max}}{\omega} \leq 0{,}5m$

$U_{max} \leq 2m/s$, ή $\boxed{U_{0,max} = 2\, m/s}$

3.4 $\varphi = 30^\circ$, $K = 200N/m$
$m_1 = 2\,kg$, $m_2 = 3\,kg$
$R_1 = 0{,}1m$, $R_2 = 0{,}2m$
$m_3 = 5\,kg$

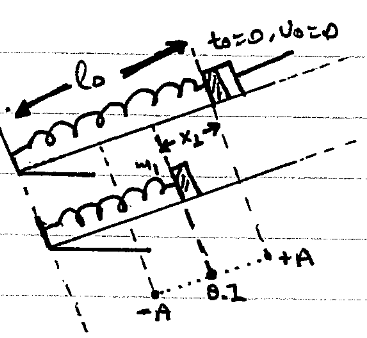


A. Το σύστημα ισορροπεί:

- Σώμα m_3: $\Sigma F = 0 \Rightarrow T_3 - W_3 = 0 \Rightarrow T_3 = m_3 g \Rightarrow \underline{T_3 = 50N\ (=T_3')}$
- Τροχαλία: $\Sigma\tau = 0 \Rightarrow T_2' \cdot R_2 - T_3' \cdot R_1 = 0 \Rightarrow \underline{T_2' = 25N\ (=T_2)}$
- Σώματα m_1, m_2: $\Sigma F_x = 0 \Rightarrow T_2 - W_{1,2(x)} - F_{ελ} = 0 \Rightarrow F_{ελ} = T_2 - (m_1 + m_2) g\,\eta\mu\varphi$

$\Rightarrow \boxed{F_{ελ} = 0}$, δηλαδή το ελατήριο βρίσκεται στο φυσικό του μήκος.

B.



- Θ.Ι.(m_1): $\Sigma F_x = 0 \Rightarrow F_{ελ} - W_{1x} = 0 \Rightarrow$

$\Rightarrow F_{ελ} = m_1 g\,\eta\mu\varphi \Rightarrow \underline{x_1 = 0{,}05m\ (=A)}$

- $\omega = \sqrt{\frac{K}{m_1}} \Rightarrow \underline{\omega = 10\,rad/s}$
- $x = A\eta\mu(\omega t + \varphi_0) \xRightarrow[x=A]{t_0=0} \eta\mu\varphi_0 = 1 = \eta\mu\frac{\pi}{2}$ ή $\varphi_0 = \frac{\pi}{2} rad$

- $F = -D \cdot x \xRightarrow{D=K} F = -K \cdot A\eta\mu(\omega t + \varphi_0) \Rightarrow \boxed{F = -10\eta\mu\left(10t + \frac{\pi}{2}\right)}$, (S.I.)

ΚΕΦΑΛΑΙΟ 3ο - ΘΕΜΑ 4ο

Γ. • $\frac{dK}{dt} = \Sigma F \cdot u = -k \cdot x \cdot u$ (1)

• $x = A ημ(ωt+φ_0) \xrightarrow{t=0,025π\,s} x = 0,05 ημ\left(\frac{π}{4}+\frac{π}{2}\right) \Rightarrow \underline{x = 0,025\sqrt{2}\ m}$

• $u = ωA συν(ωt+φ_0) \xrightarrow{t} u = 0,5 συν\left(\frac{π}{4}+\frac{π}{2}\right) \Rightarrow \underline{u = -0,25\sqrt{2}\ m/s}$

$$(1) \Rightarrow \frac{dK}{dt} = -200 \cdot 0,025\sqrt{2} \cdot (-0,25\sqrt{2})\ J/s \Rightarrow \boxed{\frac{dK}{dt} = 2,5\ J/s}$$

Δ. • Για το σύστημα τροχαλίας, σώματος m_2, σώματος m_3 ισχύει: ως προς τον άξονα περιστροφής της τροχαλίας:

$\left.\begin{matrix}|τ_{w_3}| = 5\ N\cdot m \\ |τ_{w_{2x}}| = 3\ N\cdot m\end{matrix}\right\} \Rightarrow |τ_{w_3}| > |τ_{w_{2x}}|$, δηλαδή η τροχαλία θα στραφεί δεξιόστροφα.

• Σώμα m_3: $\Sigma F = m_3 α_{cm} \Rightarrow w_3 - T_3 = m_3 α_{cm} \Rightarrow T_3 = m_3 g - m_3 α_{cm} \xrightarrow{α_{cm} = α_{γων} R_1 = 1\,m/s^2} \underline{T_3 = 45\ N}\ (=T_3')$

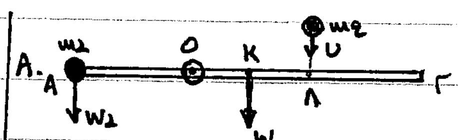
• Σώμα m_2: $\Sigma F_x = m_2 α_{cm}' \Rightarrow T_2 - W_{2x} = m_2 α_{cm}' \xrightarrow{α_{cm}' = α_{γων} R_2 = 2\,m/s^2}$
$\Rightarrow T_2 = m_2 g ημφ + m_2 α_{cm}' \Rightarrow \underline{T_2 = 21\ N}\ (=T_2')$

• Τροχαλία: $T_3' R_1 - T_2' R_2 = I α_{γων} \Rightarrow \boxed{I = 0,03\ kg\,m^2}$

Ε. $Π\% = \frac{ΔU_2}{W_{w_3}} \cdot 100\% = \frac{m_2 g \cdot S_2 \cdot ημφ}{m_3 g \cdot S_1} \cdot 100\% = \frac{m_2 \cdot R_2 \cdot \cancel{θ} \cdot ημφ}{m_3 \cdot R_1 \cdot θ} \cdot 100\%$

ή $Π\% = \frac{m_2 R_2 \cdot ημφ}{m_3 R_1} \cdot 100\% \Rightarrow \boxed{Π\% = 60\%}$

3.5 $M = 2\ kg$, $L = 3\ m$, $m_1 = 1\ kg$



• Το σύστημα ράβδου–σώματος m_1 ισορροπεί άρα:
$\Sigma τ_{(O)} = 0 \Rightarrow w_1 \cdot (AO) - W \cdot (OK) = 0$

$\xrightarrow{(OK) = \frac{L}{2} - (AO)} m_1 g \cdot (AO) - Mg\left[\frac{L}{2} - (AO)\right] = 0 \xrightarrow{S.I.} 10(AO) - 30 + 20(AO) = 0$

$\Rightarrow \boxed{(AO) = 1\ m}$

Β. 1) • Α.Δ.Στροφορμής: $\vec{L}_{αρχ} = \vec{L}_{τελ}$

$m_2 \cdot u \cdot (OΛ) = \left[I_O^{ράβδου} + m_1 (AO)^2 + m_2 (OΛ)^2\right] \cdot ω$

$\xrightarrow{θ.Steiner:\ I_O^{ράβδου} = I_{cm} + M(OK)^2} u = \left(\frac{1}{12} \cdot 2 \cdot 9 + 2 \cdot 0,25 + 1 \cdot 1^2 + 1 \cdot 1^2\right) \cdot 9\ m/s \Rightarrow$

$\Rightarrow \boxed{u = 36\ m/s}$

2) $E_{απωλειών} = K_{αρχ} - K_{τελ} = \frac{1}{2} m_2 υ^2 - \frac{1}{2} I_O ω^2 \Rightarrow$

$E_{απωλειών} = \frac{1}{2} m_2 υ^2 - \frac{1}{2}\left[\frac{1}{12}ML^2 + M(OK)^2 + m_1(AO)^2 + m_2(OΛ)^2\right] ω^2 \Rightarrow$

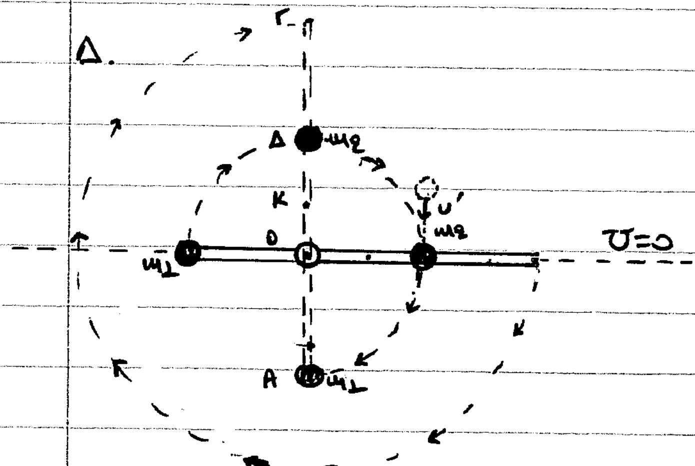
$E_{απωλειών} = \frac{1}{2}\cdot 1 \cdot 36^2 - \frac{1}{2}\left(\frac{1}{12}2\cdot 9 + 2\cdot 0{,}25 + 1\cdot 1^2 + 1\cdot 1^2\right]\cdot 9^2 \Rightarrow$

$E_{απωλειών} = (648 - 162)J \Rightarrow \boxed{E_{απωλ.} = 486\ J}$

Γ) $Στ = I_O \cdot a_{γων} \Rightarrow -w_1\cdot(AO) + w\cdot(OK) + w_2(OΛ) = I_O\ a_{γων} \Rightarrow$

$\Rightarrow a_{γων} = \dfrac{Mg\cdot(OK) + m_2 g(OΛ) - m_1 g(AO)}{\frac{1}{12}ML^2 + M(OK)^2 + m_1(AO)^2 + m_2(OΛ)^2} \Rightarrow \boxed{a_{γων} = 2{,}5\ rad/s^2}$

Δ.



1) • Α.Δ.Μ.Ε.: $E_{μηχ}^{αρχ} = E_{μηχ}^{τελ} \Rightarrow$

$K_{αρχ} + U_{αρχ} = K_{τελ} + U_{τελ} \Rightarrow$

$\Rightarrow \frac{1}{2} I_O ω'^2 = -m_1 g(AO) + m_2 g(OΔ) + Mg(OK)$

$\Rightarrow ω' = \sqrt{5}\ rad/s$

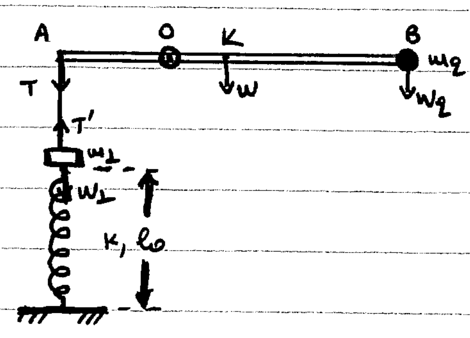
• Α.Δ. Στροφορμής: $\vec{L}_{αρχ} = \vec{L}_{τελ}$

$m_2 υ'(OΔ) = I_O \cdot ω' \Rightarrow \boxed{υ' = 4\sqrt{5}\ m/s}$

2) $\boxed{\frac{dL}{dt} = Στ = 0}$

3.6 $\ell = 3m$, $M = 12kg$
$m_1 = 10kg$
$K = 4000\ N/m$, $(OB) = 2m$
το ελατήριο στο φυσικό του μήκος ℓ_0

A.



• Το σύστημα ισορροπεί

▪ Σώμα m_1: $ΣF = 0 \Rightarrow$
$T' - w_1 = 0 \Rightarrow$
$T' = m_1 g = 100N (=T$

▪ Ράβδος: $Στ_{(O)} = 0 \Rightarrow T\cdot(AO) - w(OK) - w_2(OB) = 0 \Rightarrow T\cdot[\ell - (OB)] = Mg\left[(OB) - \frac{\ell}{2}\right]$
$+ m_2 g(OB) \overset{Σ.Ι.}{\Rightarrow} 100\cdot 1 = 120 \cdot 0{,}5 + 20 m_2 \Rightarrow \boxed{m_2 = 2\ kg}$

B. 1) $I_O = I_O^{ράβδου} + I_O^{m_2}$ (1)

Θεώρημα Steiner: $I_O^{ράβδου} = I_{cm} + M(OK)^2 = \frac{1}{12}M\ell^2 + M(OK)^2 \Rightarrow$

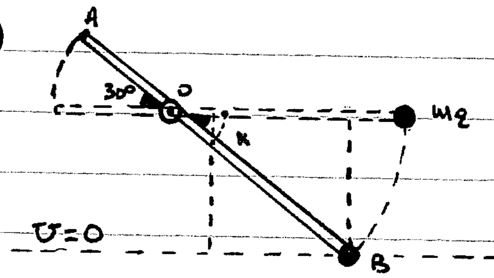
$\xrightarrow{(OK) = (OB) - \frac{\ell}{2} = 0{,}5m} I_O^{ράβδου} = 12\ kg\,m^2$

(1) $\Rightarrow I_O = I_O^{ράβδου} + m_2 (OB)^2 \Rightarrow I_O = (12 + 2\cdot 2^2)\, kg\,m^2 \Rightarrow \boxed{I_O = 20\, kg\,m^2}$

· $\Sigma\tau = I_O \cdot a_{γων} \Rightarrow w\cdot(OK) + w_2 (OB) = I_O \cdot a_{γων} \xrightarrow{(OK)=(OB)-\frac{\ell}{2}=0,5m}$

$\Rightarrow \boxed{a_{γων} = 5\, rad/s^2}$

2)



· <u>Α.Δ.Μ.Ε:</u> $E_{μηχ}^{αρχ} = E_{μηχ}^{τελ} \Rightarrow$

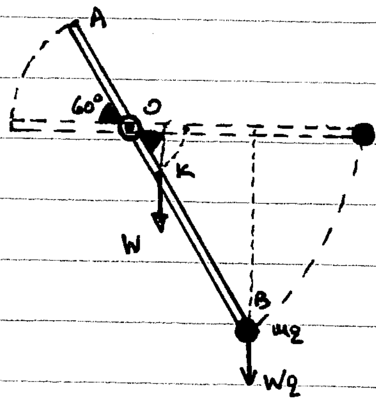
$K_{αρχ} + U_{αρχ} = K_{τελ} + U_{τελ} \Rightarrow$

$Mg(OB)ημ30° + m_2 g(OB)ημ30° = \frac{1}{2} I_O ω^2 +$

$Mg(BK)ημ30° \overset{S.I.}{\Rightarrow} 120 + 20 = 10ω^2 + 90 \Rightarrow$

$\Rightarrow \boxed{ω = \sqrt{5}\, rad/s}$

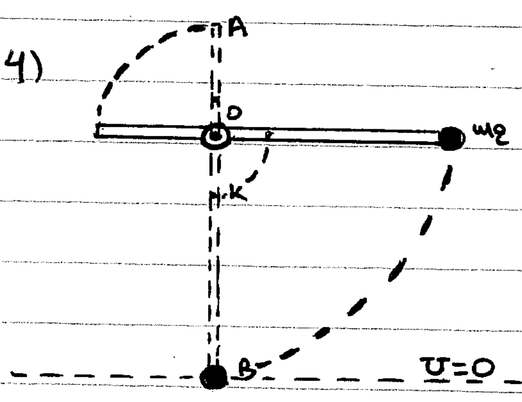
3)



· $\frac{dL}{dt} = \Sigma\tau = W\cdot(OK)συν60° + w_2(OB)συν60° \Rightarrow$

$\Rightarrow \frac{dL}{dt} = (30 + 20)\, kg\,m^2/s^2 \Rightarrow \boxed{\frac{dL}{dt} = 50\, kg\,m^2/s^2}$

4)



· <u>Α.Δ.Μ.Ε:</u> $E_{μηχ}^{αρχ} = E_{μηχ}^{τελ} \Rightarrow$

$K_{αρχ} + U_{αρχ} = K_{τελ} + U_{τελ} \Rightarrow$

$Mg(OB) + m_2 g(OB) = K + Mg(KB) \Rightarrow$

$\Rightarrow \boxed{K = 100\, J}$

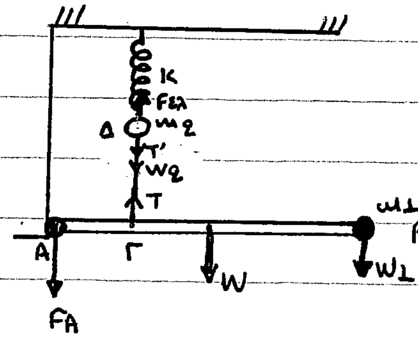
Γ. · Θ.Ι. (m_1): $\Sigma F = 0 \Rightarrow F_{ελ} - w_1 = 0 \Rightarrow k\cdot y_1 = m_1 g \Rightarrow \underline{y_1 = 0,025m\ (=A)}$

· $ω = \sqrt{\frac{k}{m_1}} = 20\, rad/s$

· $y = Aημ(ωt + φ_0) \xrightarrow[y=A]{t_0=0} A = Aημφ_0 \Rightarrow ημφ_0 = 1 = ημ\frac{π}{2}$ ή $φ_0 = \frac{π}{2}\, rad$

Άρα $\boxed{y = 0,025ημ(20t + \frac{π}{2})}$, (S.I.)

3.7 $L=1m$, $M=3kg$
$m_1=1kg$
$m_2=4kg$, $k=400\frac{N}{m}$
$(A\Gamma)=0{,}25m$



Α. • Το σύστημα ράβδος-σώμα m_1 ισορροπεί:

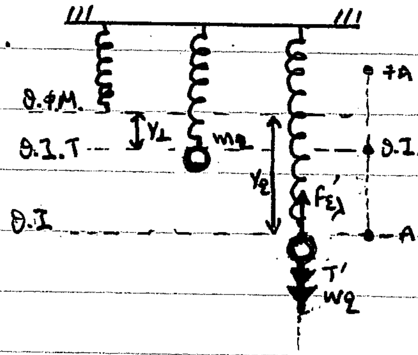
$\Sigma\tau_{(A)}=0 \Rightarrow T\cdot(A\Gamma)-W\frac{L}{2}-W_1L$

$\Rightarrow \boxed{T=100N}$

$\Sigma F_y=0 \Rightarrow T-F_A-W-W_1=0$

$\Rightarrow \boxed{F_A=60N}$

Β.



Το σφαιρίδιο m_2 θα φτάσει από τη $y=-A$ στη $y=+A$ για πρώτη φορά σε χρόνο $\Delta t=\frac{T}{2}$

$T=2\pi\sqrt{\frac{m_2}{k}}=0{,}2\pi s$ $\Big\} \Rightarrow \boxed{\Delta t=0{,}314s}$

Γ. • Θ.Ι.Τ.: $\Sigma F=0 \Rightarrow F_{ελ}-W_2=0 \Rightarrow k\cdot y_1=m_2g \Rightarrow \underline{y_1=0{,}1m}$

• Θ.Ι.: $\Sigma F=0 \Rightarrow F_{ελ}'-T'-W_2=0 \xrightarrow{T'=T=100N} k\cdot y_2=T'+m_2g \Rightarrow \underline{y_2=0{,}35m}$

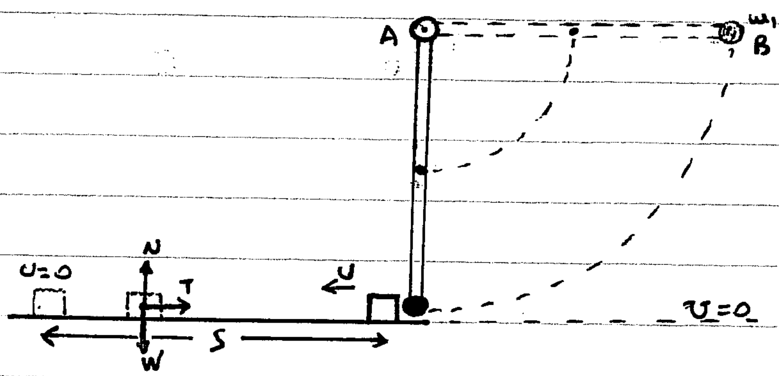
Άρα $A=y_2-y_1 \Rightarrow \underline{A=0{,}25m}$ (αφού $t_0=0$, $y=-A$, $v_0=0$).

• $y=A\eta\mu(\omega t+\varphi_0) \xrightarrow[y=-A]{t_0=0} -A=A\eta\mu\varphi_0 \Rightarrow \eta\mu\varphi_0=-1=\eta\mu\frac{3\pi}{2}$ ή $\underline{\varphi_0=\frac{3\pi}{2}rad}$

• $\omega=\frac{2\pi}{T} \Rightarrow \underline{\omega=10\,rad/s}$

$\boxed{y=0{,}25\eta\mu\left(10t+\frac{3\pi}{2}\right)}$, (S.I.)

Δ.



• Για την κίνηση της ράβδου με το σώμα m_1 θα εφαρμόσουμε:

$\underline{Α.Δ.Μ.Ε.}$: $E_{MHX}^{αρχ}=E_{MHX}^{τελ} \Rightarrow K_{αρχ}+U_{αρχ}=K_{τελ}+U_{τελ} \Rightarrow$

$\Rightarrow Mg L+m_1gL=\frac{1}{2}I_A\omega^2+Mg\frac{L}{2} \Rightarrow \frac{1}{2}I_A\omega^2=Mg\frac{L}{2}+m_1gL$ (1)

$I_A=I_A^{ραβδου}+I_A^{m_1}$

θ.Steiner: $I_A^{ραβδου}=I_{cm}+M\frac{L^2}{4}=\frac{1}{3}ML^2$ $\Big\} \Rightarrow I_A=\frac{1}{3}ML^2+m_1L^2 \Rightarrow \underline{I_A=2kgm^2}$

① $\Rightarrow$ $\omega = 5\ rad/s$

• Για την κρούση θα εφαρμόσουμε:

Α.Δ. Στροφορμής: $\vec{L}_{αρχ} = \vec{L}_{τελ}$

$$I_A \cdot \omega = m u L + I_A \frac{\omega}{5} \Rightarrow \boxed{u = 4\ m/s}$$

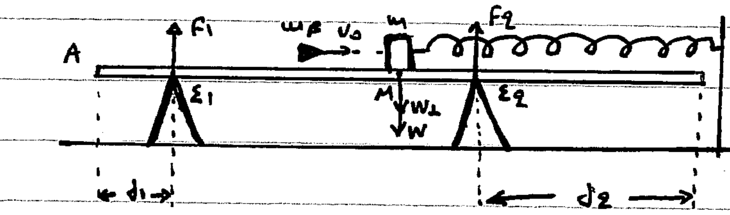
1) $$Π\% = \frac{\frac12 I_A\omega^2 - \left(\frac12 m u^2 + \frac12 I_A \frac{\omega^2}{25}\right)}{\frac12 I_A \omega^2}\cdot 100\% \Rightarrow Π\% = \frac{25-16-1}{25}\cdot 100\%$$

$$\Rightarrow \boxed{Π\% = 32\%}$$

2) ΘΜΚΕ: $\Sigma W = \Delta K \Rightarrow W_T = K_{τελ} - K_{αρχ} \Rightarrow -T\cdot S = -\frac12 m u^2 \Rightarrow$

$$\xrightarrow[T=\mu N]{\Sigma F_y = 0 \Rightarrow N = mg = 20N} \mu\cdot 20\cdot 1,6 = \frac12 \cdot 2\cdot 16 \Rightarrow \boxed{\mu = 0,5}$$

3.8 $L = 8m$, $M = 5kg$, $d_1 = 1m$, $d_2 = 3m$, $m = 3kg$, $k = 100\frac{N}{m}$, $m_B = 1kg$, u_0



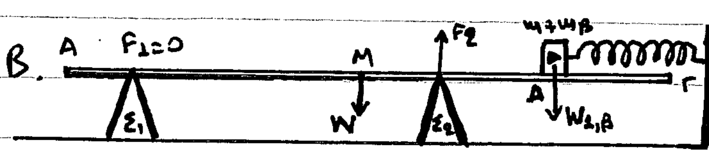
Α. Το σύστημα ράβδος - σώμα m ισορροπεί:

• $\Sigma\tau_{(Σ_1)} = 0 \Rightarrow -W\cdot(Σ_1 M) - W_1\cdot(Σ_1 M) + F_2\cdot(Σ_1Σ_2) = 0 \Rightarrow$

$$\Rightarrow F_2\cdot(L-d_1-d_2) = Mg\left(\frac L2 - d_1\right) + mg\left(\frac L2 - d_1\right) \Rightarrow$$

$$\Rightarrow \boxed{F_2 = 60N}$$

• $\Sigma F = 0 \Rightarrow F_1+F_2-W-W_1 = 0 \Rightarrow F_1 = Mg+mg-F_2 \Rightarrow \boxed{F_1 = 20N}$

Β. 

Έστω ότι το συσσωμάτωμα όταν φτάσει στη θέση Δ η ράβδος είναι έτοιμη να ανατραπεί, δηλαδή ισχύει $F_1 = 0$. Τότε έχουμε:

$$\Sigma\tau_{(Σ_2)}=0 \Rightarrow W\cdot(MΣ_2) - W_{1,B}\cdot(Σ_2Δ) = 0 \Rightarrow Mg\left(\frac L2 - d_2\right) = (m+m_B)g(Σ_2Δ) \Rightarrow$$

$$\Rightarrow (Σ_2Δ) = 1,25m$$ ή $x = 1,75m$ από το Γ.

Γ. • Αν το συσσωμάτωμα ταλαντώνει με τέτοιο πλάτος ώστε οριακά να μην ανατρέπεται η ράβδος τότε ισχύει:

$A = (M\Delta) = (M\Sigma_2) + (\Sigma_2\Delta) = \left(\frac{L}{2} - d_2\right) + (\Sigma_2\Delta)$ ή $\boxed{A = 2,25m}$

• Η ταχύτητα που έχει το συσσωμάτωμα μετά την κρούση θα αντιστοιχεί στην υ_{max} ταλάντωσης, αφού η κρούση γίνεται στη Θ.Ι. ταλάντωσης.

$$\left.\begin{array}{l} \upsilon_{max} = \omega A \\ \omega = \sqrt{\frac{k}{m+m_\beta}} \Rightarrow \omega = 5\, rad/s \end{array}\right\} \Rightarrow \underline{\upsilon_{max} = 11,25\, m/s \ (=\upsilon_\sigma)}$$

• κρούση ανελαστική (πλαστική)

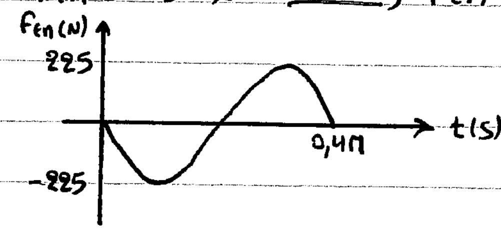
Α.Δ. Ορμής: $\vec{p}_{\alpha\rho\chi} = \vec{p}_{\tau\varepsilon\lambda}$

$$m_\beta \cdot \upsilon_0 = (m+m_\beta)\cdot \upsilon_\sigma \xrightarrow{\upsilon_\sigma = \upsilon_{max}} \boxed{\upsilon_0 = 45\, m/s}$$

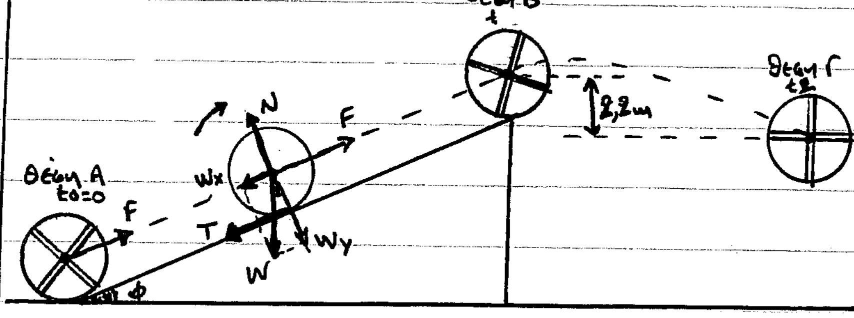
Δ. • $x = A\eta\mu(\omega t + \varphi_0) \xrightarrow[x=0]{t_0=0} 0 = A\eta\mu\varphi_0 \Rightarrow \eta\mu\varphi_0 = 0 = \eta\mu 0 \Rightarrow \left| \begin{array}{l} \varphi_0 = 2k\pi + 0 \xrightarrow{k=0} \\ \varphi_0 = 2k\pi + \pi - 0 \end{array}\right.$

$$\Rightarrow \left\{ \begin{array}{ll} \varphi_0 = 0, & \upsilon > 0 \\ \varphi_0 = \pi, & \upsilon < 0 \end{array}\right. \quad \text{άρα } \underline{\varphi_0 = 0}$$

• $F_{\varepsilon\pi} = -D\cdot x \xrightarrow{D=k} F_{\varepsilon\pi} = -k\cdot A\eta\mu(\omega t + \varphi_0) \Rightarrow \underline{F = -225\eta\mu 5t}$, (S.I.)



3.9 $\varphi = 30°$, $\upsilon_0 = 0$
$M_\Sigma = 6kg$, $R = 1m$
$\ell = 2m$, $m_p = 3kg$
$F = 100N$
$N = \frac{12,5}{\pi}$ περιστρ.



A. $I_o = I_o^{δακτυλίου} + 2 I_o^{ράβδων}$ ①

$I_o^{δακτυλίου} = m_1 R^2 + m_2 R^2 + \ldots + m_v R^2 = (m_1 + m_2 + \ldots + m_v) R^2 = MR^2 = 6\,kg\,m^2$

$I_o^{ράβδου} = I_{cm} = \frac{1}{12} m \ell^2 = 1\,kg\,m^2$

Άρα ① $\Rightarrow \boxed{I_o = 8\ kg \cdot m^2}$

B. Ο τροχός εκτελεί σύνθετη κίνηση:

- $\Sigma \tau = I_o \cdot a_{γων} \xrightarrow{a_{γων} = a_{cm}/R} T \cdot R = I_o \cdot \frac{a_{cm}}{R} \Rightarrow T = 8 a_{cm}$ (S.I.) ②
- $\Sigma F = m_{ολ} \cdot a_{cm} \Rightarrow F - W_x - T = (M + 2m) a_{cm} \Rightarrow F - (M+2m) g\, ημφ - T =$
$= (M + 2m) a_{cm} \xrightarrow{②} 100 - 12 \cdot 10 \cdot \frac{1}{2} - 8 a_{cm} = 12 a_{cm} \Rightarrow$
$\Rightarrow \underline{a_{cm} = 2\,m/s^2}$

② $\Rightarrow \boxed{T = 16N}$

Γ. $\left(\frac{dK}{dt}\right)_{στρ.} = \Sigma \tau \cdot \omega = I_o \cdot a_{γων} \cdot \omega$ ③

- $a_{γων} = \frac{a_{cm}}{R} = \underline{2\,rad/s}$
- $\Delta\theta = N \cdot 2\pi \Rightarrow \underline{\Delta\theta = 25\,rad}$

$\Delta\theta = \frac{1}{2} \cdot a_{γων} t_1^2 \Rightarrow \underline{t_1 = 5s}$

$\omega = a_{γων} t_1 \Rightarrow \omega = 10\,rad/s$

③ $\Rightarrow \boxed{\left(\frac{dK}{dt}\right)_{στρ} = 160\ J/s}$

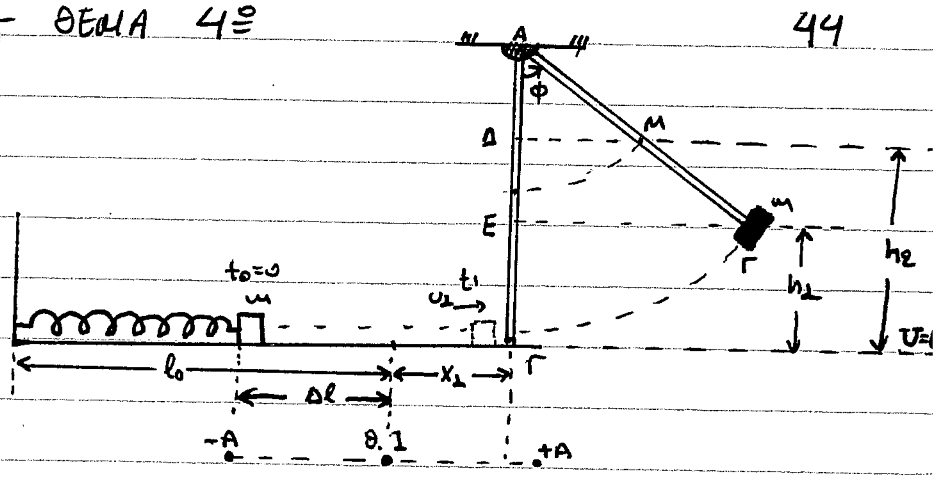
Δ. A.Δ.Μ.Ε: $E_{ΜΗΧ}^{αρχ} = E_{ΜΗΧ}^{τελ} \Rightarrow K_{αρχ} + U_{αρχ} = K_{τελ} + U_{τελ} \Rightarrow$

$\frac{1}{2} m_{ολ} v_{cm}^2 + \frac{1}{2} I_o \omega^2 + m_{ολ} g h = \frac{1}{2} m_{ολ} v_{cm}'^2 + \frac{1}{2} I_o \omega^2 \xrightarrow[\omega = σταθ]{\text{ομαλή κυκλ. κίνηση}}$

$\Rightarrow \frac{1}{2}(M+2m) v_{cm}^2 + (M+2m) g h = \frac{1}{2}(M+2m) v_{cm}'^2 \xrightarrow{v_{cm} = \omega R = 10 m/s}$

$\Rightarrow \boxed{v'_{cm} = 12\ m/s}$

3.10 $m = 1\,kg$
$k = 400\,\frac{N}{m}$, $\Delta \ell = 1m$
$M = 3\,kg$, $\ell = 1m$



A. • Η αρχική απομάκρυνση Δℓ από τη θέση ισορροπίας του σώματος m (που συμπίπτει με τη θέση φυσικού μήκους του ελατηρίου) τη στιγμή που θα το αφήσαμε ελεύθερο ($t_0=0$, $\upsilon_0=0$) θα αποτελέσει το πλάτος ταλάντωσής του, $\underline{A=\Delta\ell=1m}$.

• $k = m\omega^2 \Rightarrow \omega = \sqrt{\frac{k}{m}} \Rightarrow \underline{\omega = 20\ rad/s}$

• $x = A\eta\mu(\omega t + \varphi_0) \xrightarrow[x=-A]{t_0=0} -A = A\eta\mu\varphi_0 \Rightarrow \eta\mu\varphi_0 = -1 = \eta\mu\frac{3\pi}{2}$ ή $\underline{\varphi_0 = \frac{3\pi}{2}\,rad}$

Άρα $\boxed{x = \eta\mu(20t + \frac{3\pi}{2})}$ (1) (S.I.)

B. • (1) $\xrightarrow{x_1=\frac{\sqrt{3}}{2}m} \frac{\sqrt{3}}{2} = \eta\mu(20t + \frac{3\pi}{2}) \Rightarrow \eta\mu(20t + \frac{3\pi}{2}) = \eta\mu\frac{\pi}{3} \Rightarrow$

$\Rightarrow \begin{cases} 20t + \frac{3\pi}{2} = 2k\pi + \frac{\pi}{3} \xrightarrow{k=1} 20t = \frac{7\pi}{3} - \frac{3\pi}{2} \Rightarrow \boxed{t_1 = \frac{\pi}{24}\,s} \\ 20t + \frac{3\pi}{2} = 2k\pi + \pi - \frac{\pi}{3} \end{cases}$

• $\upsilon = \omega A \cdot \sigma\upsilon\nu(\omega t + \varphi_0) \Rightarrow \upsilon = 20\sigma\upsilon\nu(20t + \frac{3\pi}{2}) \xrightarrow{t_1=\frac{\pi}{24}s} \upsilon_1 = 20\sigma\upsilon\nu(\frac{5\pi}{6} + \frac{3\pi}{2})$

$\Rightarrow \upsilon_1 = 20\sigma\upsilon\nu\frac{7\pi}{3} \Rightarrow \upsilon_1 = 20\sigma\upsilon\nu(2\pi + \frac{\pi}{3}) \Rightarrow \boxed{\upsilon_1 = 10\,m/s}$

Γ. • Κρούση σώματος m με ράβδο ΑΓ.

Α.Δ. Στροφορμής: $\vec{L}_{\alpha\rho\chi} = \vec{L}_{\tau\varepsilon\lambda}$

$m\upsilon_1 L = I_A \cdot \omega \Rightarrow \omega = \frac{m\upsilon_1 L}{I_A}$ (2)

• $I_A = I_A^{\rho\alpha\beta\delta o\upsilon} + I_A^{m}$

Θεώρημα Steiner: $I_A^{\rho\alpha\beta\delta o\upsilon} = I_{cm} + M\frac{L^2}{4} = \frac{1}{12}ML^2 + M\frac{L^2}{4}$ ή $I_A^{\rho\alpha\beta\delta o\upsilon} = \frac{ML^2}{3}$

$I_A^{m} = mL^2$

$\Rightarrow I_A = \frac{1}{3}ML^2 + mL^2 \Rightarrow \underline{I_A = 2\,kg\,m^2}$

• Άρα (2) $\Rightarrow \boxed{\omega = 5\,rad/s}$

Δ. · Α.Δ.Μ.Ενέργειας: $E_{MHX}^{αρχ} = E_{MHX}^{τελ} \Rightarrow K_{αρχ} + U_{αρχ} = K_{τελ} + U_{τελ} \Rightarrow$

$\Rightarrow Mg\frac{L}{2} + \frac{1}{2} I_A \omega^2 = mgh_1 + Mgh_2$ (3)

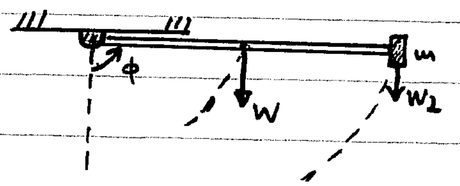
· $\overset{\Delta}{AE\Gamma}$: $συνφ = \frac{(AE)}{(A\Gamma)} \Rightarrow (AE) = Lσυνφ$, $h_1 = L - (AE) \Rightarrow h_1 = L - Lσυνφ$

· $\overset{\Delta}{A\Delta M}$: $συνφ = \frac{(A\Delta)}{(AM)} \Rightarrow (A\Delta) = \frac{L}{2}συνφ$, $h_2 = L - (A\Delta) \Rightarrow h_2 = L - \frac{L}{2}συνφ$

(3) $\Rightarrow Mg\frac{L}{2} + \frac{1}{2} I_A\omega^2 = mg(L - Lσυνφ) + Mg(L - \frac{L}{2}συνφ) \Rightarrow$

$\xRightarrow{S.I.} 15 + \frac{1}{2}\cdot 2 \cdot 25 = 10 - 10συνφ + 30 - 15συνφ \Rightarrow$

$\Rightarrow 25συνφ = 0 \Rightarrow συνφ = 0$ ή $\boxed{φ = \frac{\pi}{2}\ rad}$

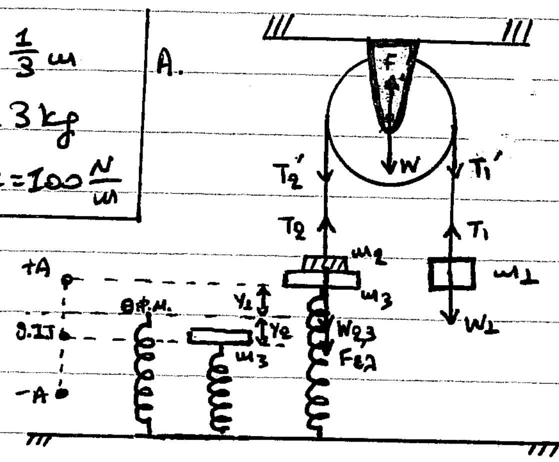


$\frac{dL}{dt} = \Sigma\tau = W\cdot\frac{L}{2} + W_1 L = Mg\frac{L}{2} + mgL$

ή $\boxed{\frac{dL}{dt} = 25\ kg\,m^2/s^2}$

3.11 $M = 8kg$, $R = \frac{1}{3}m$, $m_1 = 5kg$, $m_2 = 3kg$, $m_3 = 1kg$, $k = 100\frac{N}{m}$

A.



▶ Το σύστημα ισορροπεί:

· Σώμα m_1: $\Sigma F_y = 0 \Rightarrow T_1 - W_1 = 0 \Rightarrow T_1 = m_1 g \Rightarrow T_1 = 50N\ (= T_1')$

· Τροχαλία: $\Sigma\tau = 0 \Rightarrow T_2'R - T_1'R = 0 \Rightarrow T_2' = 50N\ (= T_2)$

$\Sigma F_y = 0 \Rightarrow F - W - T_1' - T_2' = 0$

$\Rightarrow \boxed{F = 180N}$

B. · Σώματα m_2, m_3: $\Sigma F_y = 0 \Rightarrow T_2 - W_{2,3} - F_{ελ} = 0 \Rightarrow k\cdot y_1 = T_2 - (m_2 + m_3)g$

$\Rightarrow y_1 = 0,1m$

· Θ.Ι.(m_3): $\Sigma F = 0 \Rightarrow F'_{ελ} - W_3 = 0 \Rightarrow k y_2 = m_3 g \Rightarrow y_2 = 0,1m$

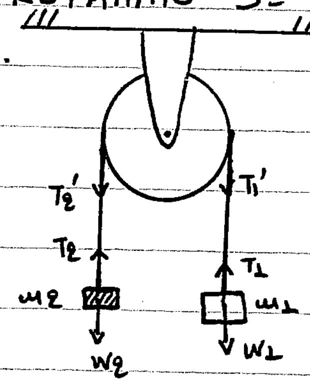
Επειδή τη στιγμή $t_0 = 0$ που τα σώματα m_2, m_3 αποκολλώνται έχουμε $υ_0 = 0$ άρα $A = y_1 + y_2 \Rightarrow A = 0,2m$

· $k = m_3\omega^2 \Rightarrow \omega = \sqrt{\frac{k}{m_3}} \Rightarrow \omega = 10\ rad/s$

· $y = Aημ(\omega t + φ_0) \xrightarrow[y=+A]{t_0=0} A = \cancel{A}\,ημ φ_0 \Rightarrow ημφ_0 = 1 = ημ\frac{\pi}{2}$ ή $φ_0 = \frac{\pi}{2}\ rad$

Άρα $\boxed{y = 0,2ημ(10t + \frac{\pi}{2})}$ (S.I.)

Γ.



• Το σώμα m_3 διέρχεται από τη θέση ισορροπίας του για πρώτη φορά μετά τη στιγμή $t_0=0$, τη στιγμή $t_1=\frac{T}{4} \xRightarrow{T=2\pi/\omega}$

$\Rightarrow \underline{t_1=0{,}05\pi\ s}$

• Σώμα m_1: $\Sigma F = m_1 \alpha_{cm} \Rightarrow W_1 - T_1 = m_1 \alpha_{cm} \Rightarrow$

$\Rightarrow T_1 = m_1 g - m_1 \alpha_{cm}$ (1)

• Σώμα m_2: $\Sigma F = m_2 \alpha_{cm} \Rightarrow T_2 - W_2 = m_2 \alpha_{cm} \Rightarrow$

$\Rightarrow T_2 = m_2 g + m_2 \alpha_{cm}$ (2)

• Τροχαλία: $\Sigma \tau = I \alpha_{\gamma\omega\nu} \xRightarrow{\alpha_{\gamma\omega\nu}=\alpha_{cm}/R} T_1' R - T_2' R = \frac{1}{2} M R^2 \frac{\alpha_{cm}}{R} \xRightarrow[T_2=T_2']{T_1=T_1'}$

$\Rightarrow T_1 - T_2 = \frac{1}{2} M \alpha_{cm} \xRightarrow[(2)]{(1)} m_1 g - m_1 \alpha_{cm} - m_2 g - m_2 \alpha_{cm} = \frac{M}{2} \alpha_{cm}$

$\Rightarrow \underline{\alpha_{cm} = \frac{5}{3}\ m/s^2}$, $\alpha_{\gamma\omega\nu} = \frac{\alpha_{cm}}{R} \Rightarrow \underline{\alpha_{\gamma\omega\nu} = 5\ rad/s^2}$

Η γωνιακή ταχύτητα της τροχαλίας τη στιγμή t_1 είναι:

$\omega = \alpha_{\gamma\omega\nu} t_1 \Rightarrow \boxed{\omega = 0{,}25\pi\ rad/s}$

Δ. $\frac{dK}{dt} = \Sigma\tau \cdot \omega = I \cdot \alpha_{\gamma\omega\nu} \cdot \omega \xRightarrow{\omega = \alpha_{\gamma\omega\nu} t_2 = 45\ rad/s} \frac{dK}{dt} = \frac{1}{2} 8 \cdot \frac{1}{9} \cdot 5 \cdot 45\ J/s$

$\Rightarrow \boxed{\frac{dK}{dt} = 100\ J/s}$

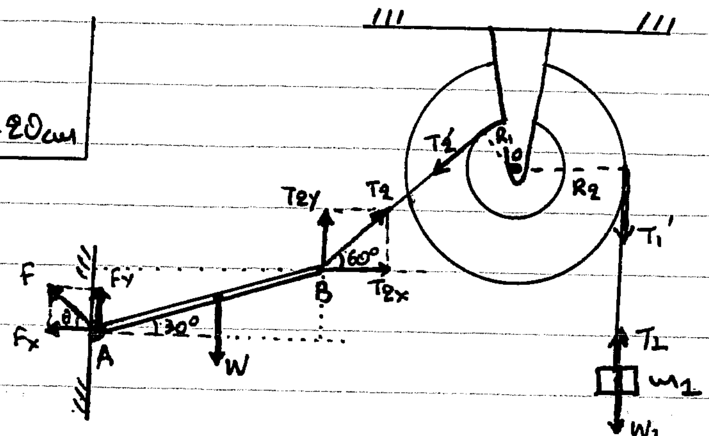
Ε. $L_{\sigma\upsilon\sigma\tau} = L_1 + L_{\tau\rho} + L_2 \Rightarrow L_{\sigma\upsilon\sigma\tau} = m_1 \upsilon_{cm} R + I\omega + m_2 \upsilon_{cm} R \Rightarrow$

$\Rightarrow L_{\sigma\upsilon\sigma\tau} = m_1 \upsilon_{cm} R + \frac{1}{2} M R^2 \omega + m_2 \upsilon_{cm} R \xRightarrow{\upsilon_{cm} = \omega R = 15\ m/s}$

$\Rightarrow \boxed{L_{\sigma\upsilon\sigma\tau} = 60\ kg\,m^2/s}$.

3.12 $m = 2\ kg$, $L = 15\ cm$

M, $R_1 = 10\sqrt{3}\ cm$, $R_2 = 20\ cm$



Α. Η ράβδος ισορροπεί:

• $\Sigma\tau_{(A)}=0 \Rightarrow T_{2y}\cdot L\cdot συν30^\circ - T_{2x}\, L\, ημ30^\circ - w\frac{L}{2}συν30^\circ = 0 \Rightarrow$

$\Rightarrow T_2 ημ60^\circ L συν30 - T_2 συν60^\circ L ημ30^\circ - mg\frac{L}{2}συν30^\circ = 0 \Rightarrow$

$\Rightarrow T_2\cdot\frac{\sqrt3}{2}\cdot\frac{\sqrt3}{2} - T_2\cdot\frac12\cdot\frac12 = 10\cdot\frac{\sqrt3}{2} \Rightarrow T_2\left(\frac34-\frac14\right)=5\sqrt3 \Rightarrow$

$\Rightarrow \boxed{T_2 = 10\sqrt3\, N}$

• $\Sigma F=0 \Rightarrow \begin{cases}\Sigma F_x=0\\ \Sigma F_y=0\end{cases} \Rightarrow \begin{cases}F_x=T_{2x}\\ F_y=w-T_{2y}\end{cases} \Rightarrow \begin{cases}F_x=T_2συν60^\circ\\ F_y=mg-T_2ημ60^\circ\end{cases} \Rightarrow \begin{cases}\underline{F_x=5\sqrt3 N}\\ \underline{F_y=5N}\end{cases}$

Άρα $F=\sqrt{F_x^2+F_y^2} \Rightarrow F=\sqrt{75+25}\,N \Rightarrow \boxed{F=10N}$

$εφθ=\frac{F_y}{F_x} \Rightarrow εφθ=\frac{\sqrt3}{3}$ ή $\underline{θ=30^\circ}$

Β. • Σώμα m_1: $\Sigma F=0 \Rightarrow T_1-W_1=0 \Rightarrow T_1=m_1 g$ ①

• Τροχαλία: $\Sigma\tau_{(0)}=0 \Rightarrow T_2'\cdot R_1 - T_1' R_2=0 \Rightarrow T_1'=\frac{R_1}{R_2}T_2' \xRightarrow{T_2=T_2'}$

$\Rightarrow \underline{T_1'=15N\ (=T_1)}$

① $\Rightarrow \boxed{m_1=1,5\ kg}$

Γ. • $\underline{Α.Δ.Μ.Ε.}$: $E_{μηχ}^{αρχ}=E_{μηχ}^{τελ} \Rightarrow K_{αρχ}+U_{αρχ}=K_{τελ}+U_{τελ} \Rightarrow$

$\Rightarrow mg\frac{L}{2}ημ30^\circ = \frac12 I_A\cdot\omega^2$ ②

• Θεώρημα Steiner: $I_A=I_{cm}+m\frac{L^2}{4} \Rightarrow I_A=\frac1{12}mL^2+\frac14 mL^2 \Rightarrow$

$\Rightarrow I_A=\frac{4mL^2}{12} \Rightarrow \underline{I_A=\frac13 mL^2}$

② $\Rightarrow mg\frac{L}{2}\cdot ημ30^\circ = \frac12\cdot\frac13 mL^2\omega^2 \Rightarrow g\cdot ημ30^\circ=\frac13 L\omega^2 \Rightarrow$

$\Rightarrow \omega^2=\frac{3\cdot g\cdot ημ30^\circ}{L} \Rightarrow \boxed{\omega=10\ rad/s}$

Δ. • Σώμα m_1: $\begin{cases}υ_{cm}=α_{cm}t\\ y=\frac12 α_{cm}t^2\end{cases} \Rightarrow \begin{cases}t=\frac{υ_{cm}}{α_{cm}}\\ h=\frac12 α_{cm}t^2\end{cases} \Rightarrow \begin{cases}t=\frac{υ_{cm}}{α_{cm}}\\ h=\frac12 α_{cm}\frac{υ_{cm}^2}{α_{cm}^2}\end{cases} \Rightarrow$

$\Rightarrow \begin{cases}t=\frac{υ_{cm}}{α_{cm}}\\ h=\frac{υ_{cm}^2}{2α_{cm}}\end{cases} \Rightarrow \begin{cases}t=\frac{υ_{cm}}{α_{cm}}\\ α_{cm}=\frac{υ_{cm}^2}{2h}\end{cases} \Rightarrow \begin{cases}\underline{t=1s}\\ \underline{α_{cm}=5m/s^2}\end{cases}$

$\Sigma F=m_1α_{cm} \Rightarrow W_1-T_1=m_1α_{cm} \Rightarrow T_1=m_1g-m_1α_{cm} \Rightarrow \underline{T_1=7,5N}$

• Τροχαλία: $\Sigma\tau=I_0α_{γων} \xRightarrow{α_{γων}=α_{cm}/R_2=25\ rad/s^2} T_1' R_2=I_0α_{γων}$

$\xRightarrow{T_1'=T_1} \boxed{I_0=0,06\ kg m^2}$

Ε. $\Pi\% = \dfrac{K_{\tau\rho}}{U_1^{\alpha\rho\chi} - U_1^{\tau\varepsilon\lambda}} \cdot 100\%$ (3)

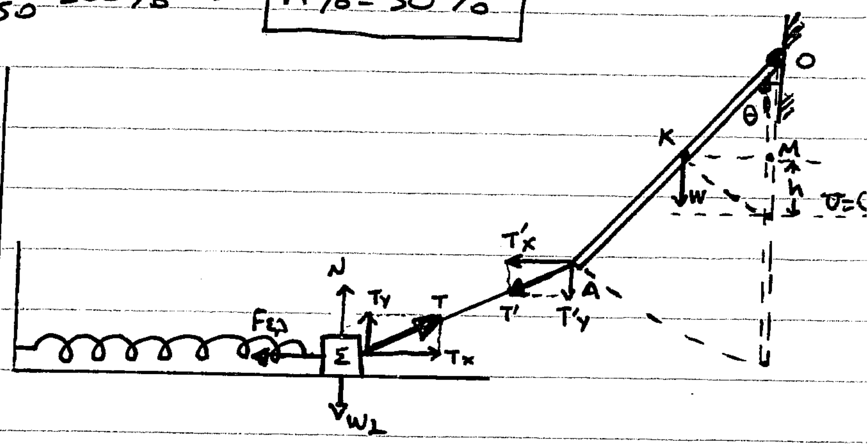
· Σώμα m_1: $y = \frac{1}{2}\alpha_{cm} t_1^2 \Rightarrow y = 10m$

· Τροχαλία: $\omega = \alpha_{\gamma\omega\nu} t_1 \Longrightarrow \omega = 50\ rad/s$

$$(3) \Rightarrow \Pi\% = \frac{\frac{1}{2} I_o \omega^2}{m_1 g y - 0} \cdot 100\% \Rightarrow \Pi\% = \frac{\frac{1}{2}\cdot 0{,}06 \cdot 2500}{1{,}5\cdot 10\cdot 10} 100\%$$

$$\Rightarrow \Pi\% = \frac{75}{150} 100\% \Rightarrow \boxed{\Pi\% = 50\%}$$

3.13 $m = 1kg$, $k = 100\ N/m$, $M = 0{,}4kg$, $\ell = 0{,}5m$, $\eta\mu\theta = 0{,}8$, $\sigma\upsilon\nu\theta = 0{,}6$, $U_{\varepsilon\lambda} = 0{,}32\ J$, $D = k$



Α. · $U_{\varepsilon\lambda} = \frac{1}{2} k x_1^2 \Rightarrow x_1 = \sqrt{\frac{2U_{\varepsilon\lambda}}{k}} \Rightarrow \underline{x_1 = 0{,}08m\ (=A)}$

· $k = m\omega^2 \Rightarrow \omega = \sqrt{\frac{k}{m}} \Rightarrow \underline{\omega = 10\ rad/s}$

· $x = A\eta\mu(\omega t + \varphi_0) \xrightarrow[x=A]{t_0=0} A = A\eta\mu\varphi_0 \Rightarrow \eta\mu\varphi_0 = 1 = \eta\mu\frac{\pi}{2}$ ή $\underline{\varphi_0 = \frac{\pi}{2} rad}$

Άρα $\boxed{x = 0{,}08\eta\mu(10t + \frac{\pi}{2})}$, (S.I.)

Β. · $\underline{Α.Δ.Μ.Ε.}$: $E_{MHX}^{\alpha\rho\chi} = E_{MHX}^{\tau\varepsilon\lambda} \Rightarrow K_{\alpha\rho\chi} + U_{\alpha\rho\chi} = K_{\tau\varepsilon\lambda} + U_{\tau\varepsilon\lambda} \Rightarrow$

$\Rightarrow Mgh = \frac{1}{2} I_o \omega^2$ (1)

· Θεώρημα Steiner: $I_o = I_{cm} + M\frac{\ell^2}{4} = \frac{1}{12}M\ell^2 + M\frac{\ell^2}{4}$ ή $\underline{I_o = \frac{1}{3}M\ell^2}$

· $\overset{\triangle}{OMK}$: $\sigma\upsilon\nu\theta = \frac{(OM)}{(OK)} \Rightarrow (OM) = \frac{\ell}{2}\sigma\upsilon\nu\theta$, $h = \frac{\ell}{2} - (OM) \Rightarrow \underline{h = \frac{\ell}{2} - \frac{\ell}{2}\sigma\upsilon\nu\theta}$

$$(1) \Rightarrow Mg\frac{\ell}{2}(1 - \sigma\upsilon\nu\theta) = \frac{1}{2}\,\frac{1}{3}M\ell^2\omega^2 \Rightarrow 5 - 3 = \frac{1}{6}0{,}5\omega^2$$

$\Rightarrow \omega = \sqrt{24}\ rad/s \Rightarrow \underline{\omega = 2\sqrt{6}\ rad/s}$

· $\upsilon_A = \omega\ell \Rightarrow \boxed{\upsilon_A = \sqrt{6}\ m/s}$

Γ. · $\underline{α.α.τ.\ (m)}$: $E = K + U \xrightarrow{K=U} E = 2U \Rightarrow \frac{1}{2}kA^2 = 2\cdot\frac{1}{2}kx^2 \Rightarrow$

$\Rightarrow x^2 = \frac{A^2}{2} \Rightarrow x = \pm\frac{A\sqrt{2}}{2}$ ή $x = \pm 0{,}04\sqrt{2}m \xrightarrow{1η\ φορά} \underline{x = 0{,}04\sqrt{2}m}$

· $\frac{dp}{dt} = \Sigma F = -D\cdot x = -k\cdot x = -100\cdot 0{,}04\sqrt{2}\ kg\,m/s^2$ ή $\boxed{\frac{dp}{dt} = -4\sqrt{2}\ kg\,m/s^2}$

Δ. · <u>Σώμα m_1:</u> $\Sigma F = 0 \Rightarrow \begin{cases} \Sigma F_x = 0 \\ \Sigma F_y = 0 \end{cases} \Rightarrow \begin{cases} T_x - F_{ελ} = 0 \\ T_y + N - w_1 = 0 \end{cases} \Rightarrow \begin{cases} T_x = k \cdot x_1 \\ N = w_1 - T_y \end{cases} \Rightarrow$

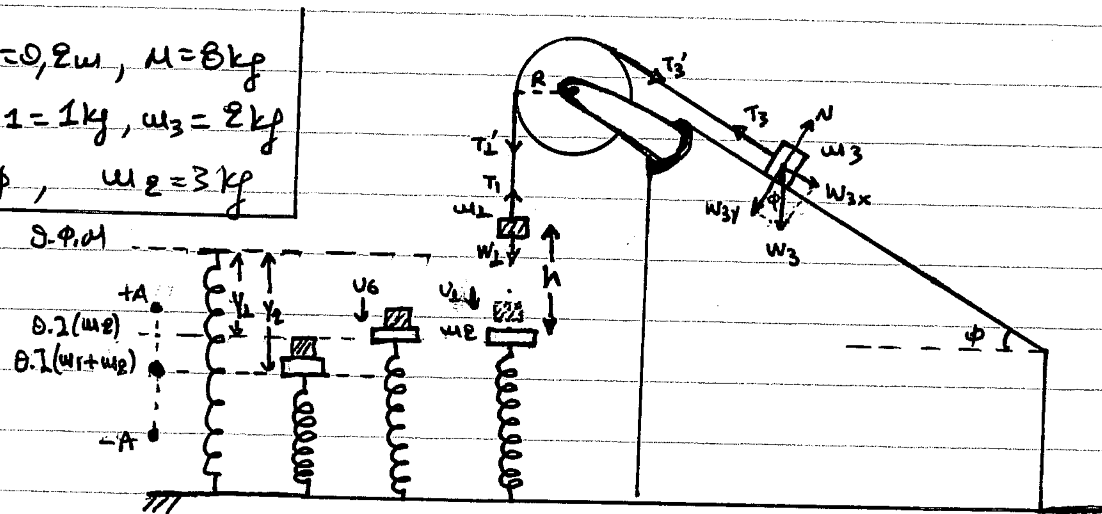
$\Rightarrow \begin{cases} \underline{T_x = 8N \ (=T'_x)} \\ N = mg - T_y \end{cases}$

· <u>Ράβδος:</u> $\Sigma\tau_{(O)} = 0 \Rightarrow T'_y\, \ell ημθ + w \cdot \frac{\ell}{2} ημθ - T'_x\, \ell συνθ = 0$

$\xrightarrow{Σ.Ζ.} T'_y \cdot 0{,}8 = 8 \cdot 0{,}6 - 4 \cdot \frac{1}{2} \cdot 0{,}8 \Rightarrow \underline{T'_y = 4N \ (=T_y)}$

Άρα $T = \sqrt{T_x^2 + T_y^2} \Rightarrow T = \sqrt{64+16}\ N \Rightarrow T = \sqrt{80}\ N$ ή $\boxed{T = 4\sqrt{5}\ N}$

3.14 $R = 0{,}2m$, $M = 8kg$
$m_1 = 1kg$, $m_3 = 2kg$
ϕ, $m_2 = 3kg$



A. Το σύστημα τροχαλία, σώματα m_1, m_3 ισορροπεί:

· Σώμα m_1: $\Sigma F_y = 0 \Rightarrow T_1 - w_1 = 0 \Rightarrow T_1 = m_1 g \Rightarrow \underline{T_1 = 10N \ (=T_1')}$

· Τροχαλία: $\Sigma\tau = 0 \Rightarrow T_1' R - T_3' \cdot R = 0 \Rightarrow \underline{T_3' = 10N \ (=T_3)}$

· Σώμα m_3: $\Sigma F_x = 0 \Rightarrow T_3 - w_{3x} = 0 \Rightarrow T_3 = m_3 g ημφ \Rightarrow ημφ = \frac{1}{2}$

ή $\boxed{\phi = \frac{\pi}{6}\ rad}$

B. · Σώμα m_1: $\Sigma F_y = m_1 \alpha_{cm} \Rightarrow w_1 - T_1 = m_1 \alpha_{cm} \Rightarrow T_1 = m_1 g - m_1 \alpha_{cm}$ (1)

· Τροχαλία: $\Sigma\tau = I \cdot \alpha_{γων} \xrightarrow{\alpha_{γων} = \alpha_{cm}/R} T_1' R = \frac{1}{2} M R^2 \cdot \frac{\alpha_{cm}}{R} \xrightarrow[(1)]{T_1' = T_1}$

$\Rightarrow m_1 g - m_1 \alpha_{cm} = \frac{1}{2} M \alpha_{cm} \Rightarrow \underline{\alpha_{cm} = 2 m/s^2}$

$\alpha_{γων} = \frac{\alpha_{cm}}{R} \Rightarrow \underline{\alpha_{γων} = 10\ rad/s^2}$, άρα $\boxed{\frac{d\omega}{dt} = \alpha_{γων} = 10\ rad/s^2}$

Γ. · Σώμα m_3: $\Sigma F_x = m_3 \alpha \Rightarrow w_{3x} = m_3 \alpha \Rightarrow \alpha = g ημφ = 5\ m/s^2$

$\upsilon = \alpha t \Rightarrow \underline{t_1 = 0{,}5 s}$

• $L = I\omega + m_1 u_{cm} R$ (2)

$\omega = a_{\gamma\omega\nu} t_1 \Rightarrow \omega = 5 \ /s$

$u_{cm} = a_{cm} t_1 \Rightarrow u_{cm} = 1 \, m/s$

Αρα (2) $\Rightarrow L = \frac{1}{2} M R^2 \omega + m_1 u_{cm} R \Rightarrow \boxed{L = 1 \, kgm^2/s}$

Δ.1) • Σωμα m_1: $y = \frac{1}{2} a_{cm} t^2 \xrightarrow{y=h} \underline{t = \sqrt{3} \, s}$

$u_1 = a_{cm} t \Rightarrow \underline{u_1 = 2\sqrt{3} \, m/s}$

• κρούση ανελαστική (πλαστική)

Α.Δ. Ορμής: $\vec{p}_{\alpha\rho\chi} = \vec{p}_{\tau\epsilon\lambda}$

$m_1 u_1 = (m_1 + m_2) u_6 \Rightarrow \underline{u_6 = \frac{\sqrt{3}}{2} \, m/s}$

$$\Pi\% = \frac{K_{\alpha\rho\chi} - K_{\tau\epsilon\lambda}}{K_{\alpha\rho\chi}} \cdot 100\% = \frac{\frac{1}{2} m_1 u_1^2 - \frac{1}{2}(m_1+m_2) u_6^2}{\frac{1}{2} m_1 u_1^2} 100\%$$

$$\Pi\% = \frac{12 - 3}{12} 100\% \Rightarrow \boxed{\Pi\% = 75\%}$$

2) • Θ.Ι.(m_2): $\Sigma F = 0 \Rightarrow F_{\epsilon\lambda} - m_2 g = 0 \Rightarrow y_1 = \frac{m_2 g}{k} \Rightarrow \underline{y_1 = 0,3m}$

• Θ.Ι. (m_1+m_2): $\Sigma F = 0 \Rightarrow F'_{\epsilon\lambda} - W_{1,2} = 0 \Rightarrow y_2 = \frac{(m_1+m_2)g}{k} \Rightarrow \underline{y_2 = 0,4m}$

• Α.Δ.Ε. Ταλαντωσης:

$E = K + U \Rightarrow \frac{1}{2} k A^2 = \frac{1}{2}(m_1+m_2) u_6^2 + \frac{1}{2} k (y_2 - y_1)^2 \Rightarrow$

$\xRightarrow{S.I.} 100 A^2 = 4 \cdot \frac{3}{4} + 100 \cdot 0,01 \Rightarrow \underline{A = 0,2m}$

• $k = (m_1+m_2)\omega^2 \Rightarrow \omega = \sqrt{\frac{k}{m_1+m_2}} \Rightarrow \underline{\omega = 5 \, rad/s}$

• $y = A \eta\mu(\omega t + \varphi_0) \xrightarrow[y = y_2 - y_1]{t=0, \, u<0} 0,1 = 0,2 \eta\mu\varphi_0 \Rightarrow \eta\mu\varphi_0 = \frac{1}{2} = \eta\mu\frac{\pi}{6} \Rightarrow$

$\Rightarrow \begin{cases} \varphi_0 = 2k\pi + \frac{\pi}{6} \\ \varphi_0 = 2k\pi + \pi - \frac{\pi}{6} \end{cases} \xRightarrow{k=0} \begin{cases} \varphi_0 = \frac{\pi}{6}, & u>0 \\ \varphi_0 = \frac{5\pi}{6}, & u<0 \end{cases}$ Αρα $\underline{\varphi_0 = \frac{5\pi}{6} \, rad}$

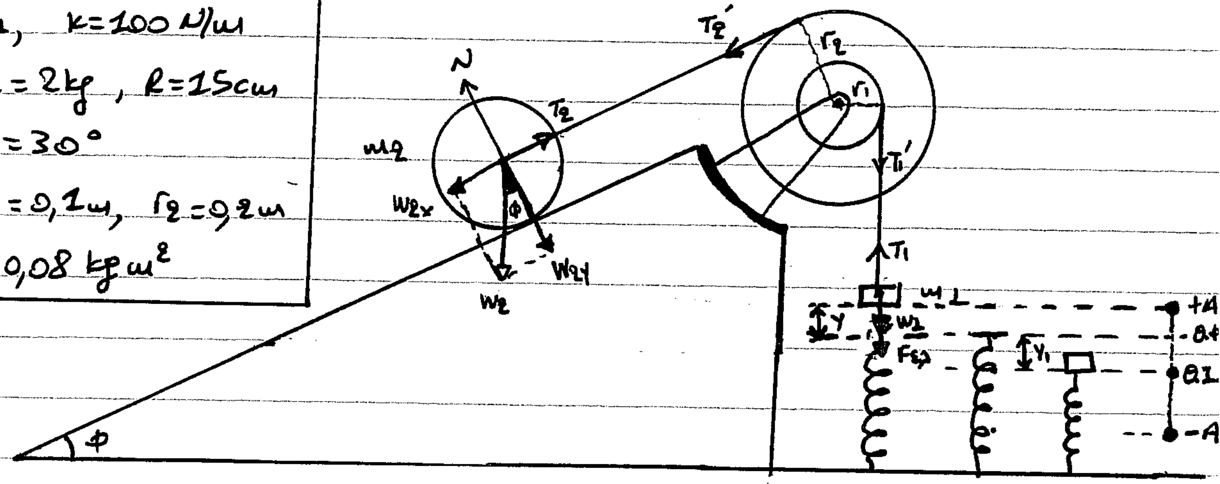
$\boxed{y = 0,2 \eta\mu\left(5t + \frac{5\pi}{6}\right)}$ (3) (S.I.)

3) • (3) $\xRightarrow{y = y_2 - y_1} 0,1 = 0,2 \eta\mu\left(5t + \frac{5\pi}{6}\right) \Rightarrow \eta\mu\left(5t + \frac{5\pi}{6}\right) = \frac{1}{2} = \eta\mu\frac{\pi}{6} \Rightarrow$

$\Rightarrow \begin{cases} 5t + \frac{5\pi}{6} = 2k\pi + \frac{\pi}{6} \xrightarrow{k=1} t_1 \\ 5t + \frac{5\pi}{6} = 2k\pi + \pi - \frac{\pi}{6} \xrightarrow{k=1} 5t_2 = 2\pi + \frac{5\pi}{6} - \frac{5\pi}{6} \Rightarrow \boxed{t_2 = 0,4\pi s \ (=T)} \end{cases}$

• $a = -\omega^2 \cdot y \xRightarrow{y = y_2 - y_1} \boxed{a = -2,5 \, m/s^2}$

3.15 m_1, $k=100\,N/m$
$m_2=2kg$, $R=15cm$
$\phi=30°$
$r_1=0{,}1m$, $r_2=0{,}2m$
$I=0{,}08\,kg\,m^2$



A. Το σύστημα ισορροπεί:

- Τροχός: $\Sigma F_x=0 \Rightarrow T_2-W_{2x}=0 \Rightarrow T_2=m_2 g\,\eta\mu\phi \Rightarrow \underline{T_2=10N\,(=T_2')}$
- Τροχαλία: $\Sigma\tau=0 \Rightarrow T_2' r_2 - T_1' r_1=0 \Rightarrow T_1'=T_2'\frac{r_2}{r_1} \Rightarrow \underline{T_1'=20N\,(=T_1)}$
- Σώμα m_1: $\Sigma F_y=0 \Rightarrow T_1-w_1-F_{ελ}=0 \Rightarrow T_1-m_1 g-ky=0 \Rightarrow m_1=\frac{T_1-ky}{g} \Rightarrow$
$\Rightarrow \boxed{m_1=1\,kg}$

B. Όταν το νήμα που συνδέει την τροχαλία με το σώμα m_1 κόβεται, ο δίσκος αρχίζει να κυλίεται χωρίς να ολισθαίνει υπό την επίδραση της στατικής τριβής T που εμφανίζεται στο σημείο επαφής του με το κεκλιμένο επίπεδο και με φορά προς τα πάνω.

- Τροχαλία: $\Sigma\tau=I a_{γων} \xrightarrow{a_{γων}=a_{cm}/r_2} T_2' r_2 = I\frac{a_{cm}}{r_2} \Rightarrow T_2'=\frac{I\cdot a_{cm}}{r_2^2}$ (1)

- Τροχός: $\Sigma\tau=I_{cm} a_{γων} \xrightarrow{a_{γων}=a_{cm}/R} T\cdot R=\frac{1}{2}m_2 R^2\frac{a_{cm}}{R} \Rightarrow T=\frac{1}{2}m_2 a_{cm}$ (2)

$\Sigma F_x=m_2\cdot a_{cm} \Rightarrow W_{2x}-T_2-T=m_2 a_{cm} \xrightarrow[T_2=T_2']{(1),(2)} m_2 g\,\eta\mu\phi-\frac{I a_{cm}}{r_2^2}-$
$-\frac{1}{2}m_2 a_{cm}=m_2 a_{cm} \Rightarrow \underline{a_{cm}=2\,m/s^2}$

Άρα $\boxed{\frac{dU_{cm}}{dt}=a_{cm}=2\,m/s^2}$

Γ. • $L=I\omega \Rightarrow \omega=\frac{L}{I} \Rightarrow \underline{\omega=30\,rad/s}$
$\omega=a_{γων} t \xrightarrow{a_{γων}=a_{cm}/r_2=10\,rad/s^2} \underline{t_1=3\,s}$

1) • $L'=I_{cm}\cdot\omega'$
$\omega'=a_{γων}' t_1 \xrightarrow{a_{γων}'=a_{cm}/R=\frac{40}{3}\frac{rad}{s^2}} \omega'=40\,rad/s$
$\Rightarrow L'=\frac{1}{2}m_2 R^2\omega' \Rightarrow \boxed{L'=0{,}9\,kg\frac{m^2}{s}}$

2) $K' = \frac{1}{2} I_{cm}\omega'^2 + \frac{1}{2} m_2 v_{cm}^2 \xrightarrow{v_{cm} = a_{cm} t_1 = 6m/s} K' = \frac{1}{2}\cdot\frac{1}{2} m_2 R^2 \omega'^2 + \frac{1}{2} m_2 v_{cm}^2$

$\xrightarrow{v_{cm} = \omega R} K' = \frac{1}{4} m_2 v_{cm}^2 + \frac{1}{2} m_2 v_{cm}^2 \Rightarrow K' = \frac{3}{4} m_2 v_{cm}^2 \Rightarrow \boxed{K' = 54 J}$

3) $\frac{dK}{dt} = \Sigma F \cdot v_{cm} + \Sigma \tau \cdot \omega \Rightarrow \frac{dK}{dt} = m_2 a_{cm} v_{cm} + I_{cm} \cdot a_{γων}\, \omega \Rightarrow$

$\Rightarrow \frac{dK}{dt} = \left(2\cdot 2\cdot 6 + \frac{1}{2}\cdot 2\cdot 0{,}15^2 \cdot \frac{40}{3}\cdot 40\right)\frac{J}{s} \Rightarrow \boxed{\frac{dK}{dt} = 36\ J/s}$

Α. • Θ.Ι.Τ: $\Sigma F = 0 \Rightarrow F_{ελ} - w_1 = 0 \Rightarrow k\cdot y_1 = m_1 g \Rightarrow y_1 = 0{,}1m$.

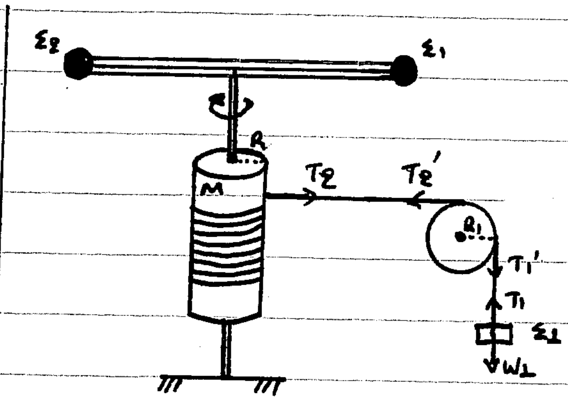
• $A = y + y_1 \Rightarrow \underline{A = 0{,}2m}$

• $k = m_1 \omega^2 \Rightarrow \omega = \sqrt{\frac{k}{m_1}} \Rightarrow \underline{\omega = 10\ rad/s}$

• $y = A ημ(\omega t + \varphi_0) \xrightarrow[y = 0{,}2m]{t_0 = 0} 0{,}2 = 0{,}2 ημ\varphi_0 \Rightarrow ημ\varphi_0 = 1 = ημ\frac{\pi}{2}$ ή $\underline{\varphi_0 = \frac{\pi}{2} rad}$

Άρα $\boxed{y = 0{,}2 ημ\left(10t + \frac{\pi}{2}\right)}$, (S.I.)

3.16 $m_1 = 2kg$ (Σ_1)
τροχαλία: $m = 2kg$, R_1
κύλινδρος: $R = 0{,}2m$, M
ράβδος: $L = 1m$
$m_2 = m_3 = 0{,}025kg$
$T_{ορ} = 25N$



Α. • Σώμα m_1: $\Sigma F = m_1 a_{cm} \Rightarrow w_1 - T_1 = m_1 a_{cm} \Rightarrow T_1 = m_1 g - m_1 a_{cm} \Rightarrow$

$\Rightarrow \boxed{T_1 = 12N}$

Β. • Η μεταφορική επιτάχυνση του σώματος m_1 ($a_{cm} = 4 m/s^2$), είναι ίση (κατά μέτρο) με την επιτρόχια επιτάχυνση της τροχαλίας και με την επιτρόχια επιτάχυνση του κυλίνδρου.

$\left.\begin{array}{l} a_{cm} = a_{τρ} = a_{κυλ} \\ a_{κυλ} = a_{γων}\cdot R \end{array}\right\} \Rightarrow \underline{a_{γων} = 20\ rad/s}$

• Το σύστημα κύλινδρος - ράβδος με τα σώματα Σ3, Σ2 περιστρέφεται με την ίδια γωνιακή ταχύτητα.

$\omega = a_{γων} t \xrightarrow{t = 1{,}5\pi s} \underline{\omega = 30\pi\ rad/s}$

• $\omega = 2\pi f \Rightarrow f = \frac{\omega}{2\pi} \Rightarrow \boxed{f = 15\,Hz}$

Γ. • Τροχαλία: $\Sigma\tau = I \cdot \alpha_{γων} \xrightarrow{\alpha_{γων} = \alpha_{cm}/R_1} T_1' R_1 - T_2' R_1 = \frac{1}{2} m R_1^2 \frac{\alpha_{cm}}{R_1} =$

$\xrightarrow{T_1' = T_1} 12 - T_2' = \frac{1}{2}\, 2 \cdot 4 \Rightarrow \underline{T_2' = 8N\,(=T_2)}$

• Σύστημα κυλίνδρου - ράβδου και σωμάτων $Σ_3, Σ_2$:

$\Sigma\tau = I \cdot \alpha_{γων} \Rightarrow T_2 \cdot R = [I_{κυλ.} + m_2 (\frac{L}{2})^2 + m_3 (\frac{L}{2})^2] \cdot \alpha_{γων}$

$\Rightarrow \frac{T_2 \cdot R}{\alpha_{γων}} = I_{κυλ.} + 2 m_2 \frac{L^2}{4} \Rightarrow I_{κυλ.} = \frac{T_2 R}{\alpha_{γων}} - m_2 \frac{L^2}{2} \Rightarrow$

$\Rightarrow \boxed{I_{κυλ.} = 0{,}0675\ kg \cdot m^2}$

Δ. • Για την κεντρομόλο δύναμη που ασκείται σε καθένα από τα σώματα m_2, m_3, ισχύει:

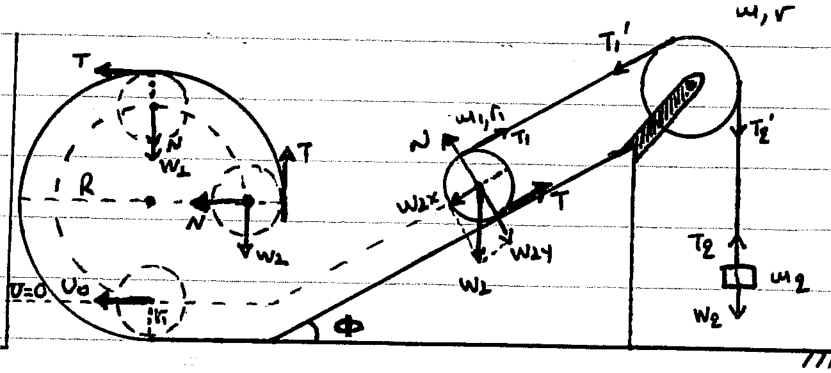
$F_κ = m_2 \cdot \frac{υ^2}{\frac{L}{2}} \xrightarrow{υ = \omega \cdot \frac{L}{2}} F_κ = m_2 \omega^2 \cdot \frac{L}{2} \xrightarrow{F_κ = \Sigma F = T_{θρ}}$

$\Rightarrow \omega = \sqrt{\frac{2 \cdot T_{θρ}}{m_2 \cdot L}} \Rightarrow \omega = \sqrt{2000}\ rad/s \Rightarrow \underline{\omega = 20\sqrt{5}\ rad/s}$

• $\omega = \alpha_{γων} t \Rightarrow \underline{t = \sqrt{5}\ s}$

• $\theta = \frac{1}{2} \alpha_{γων} t^2 \Rightarrow \boxed{\theta = 50\ rad}$

3.17 $m_1 = 6kg$, $r_1 = 0{,}3m$
$\varphi = 30°$, m_2
τροχαλία: m, r
στεφάνη: $R = 0{,}7m$
$υ_0 = 10\ m/s$



A. • Σώμα m_1: $\Sigma\tau = 0 \Rightarrow T_1 \cdot r_1 - T r_1 = 0 \Rightarrow \underline{T_1 = T}$ (1)

$\Sigma F_x = 0 \Rightarrow w_{1x} - T_1 - T = 0 \xrightarrow{(1)} m_1 g\, ημ\varphi = 2T \Rightarrow \underline{T = 15N\,(=T_1)}$

• Τροχαλία: $\Sigma\tau = 0 \Rightarrow T_1' r - T_2' r = 0 \xrightarrow{T_1' = T_1} \underline{T_2' = T_1' = 15N\,(=T_2)}$

• Σώμα m_2: $\Sigma F_y = 0 \Rightarrow T_2 - w_2 = 0 \Rightarrow T_2 = m_2 g \Rightarrow \boxed{m_2 = 1{,}5\ kg}$

B. • Η ταχύτητα του σώματος m_2 είναι ίση κατά μέτρο με την γραμμική ταχύτητα της τροχαλίας και ίση με την ταχύτητα που έχει το ανώτερο σημείο του τροχού, άρα:

$υ = 2υ_{cm} = 20\,m/s$

• Το σώμα m_2 κινείται και αυτό με σταθερή ταχύτητα

$υ = \frac{h}{t} \Rightarrow h = υ \cdot t \xrightarrow{t=0,3s} \boxed{h = 6m}$

Γ. • Στο ανώτερο σημείο της στεφάνης ισχύει για τον τροχό:

$F_K = m_1 \frac{υ^2}{R-r_1} \xrightarrow{F_K = ΣF_y} N + w_1 = m_1 \frac{υ^2}{R-r_1} \xrightarrow[N=0]{υ=υ_{op}} m_1 g = m_1 \frac{υ_{op}^2}{R-r_1} \Rightarrow$

$\Rightarrow υ_{op} = \sqrt{g(R-r_1)} \Rightarrow \boxed{υ_{op} = 2\,m/s}$

Δ. • Η ελάχιστη κινητική ενέργεια που πρέπει να έχει ο τροχός στο ανώτερο σημείο της στεφάνης είναι

$K = \frac{1}{2} m_1 υ^2 + \frac{1}{2} I ω^2 \Rightarrow K = \frac{1}{2} m_1 υ^2 + \frac{1}{2} \frac{1}{2} m_1 r_1^2 ω^2 \xrightarrow{υ=ωr}$

$\Rightarrow K = \frac{1}{2} m_1 υ^2 + \frac{1}{4} m_1 υ^2 \Rightarrow K = \frac{3}{4} m_1 υ^2 \xrightarrow[K=K_{min}]{υ=υ_{op}} K_{min} = \frac{3}{4} m_1 υ_{op}^2 \Rightarrow$

$\Rightarrow \underline{K_{min} = 18\,J}$

• Η κινητική ενέργεια με την οποία φτάνει ο τροχός στο ανώτερο σημείο της στεφάνης είναι:

<u>Α.Δ.Μ.Ε.</u>: $E_{MHX}^{αρχ} = E_{MHX}^{τελ} \Rightarrow K_{αρχ} + U_{αρχ} = K_{τελ} + U_{τελ}$

$\Rightarrow \frac{1}{2} m_1 υ_0^2 + \frac{1}{2} I ω_0^2 = K + m_1 g 2(R-r_1) \Rightarrow \frac{3}{4} m_1 υ_0^2 = K + 2 m_1 g (R-r_1)$

$\Rightarrow \underline{K = 402\,J}$

▸ Αφού $K = 402J > K_{min} = 18J$ άρα ο τροχός θα κάνει ασφαλή ανακύκλωση.

Ε. • Θα μελετήσουμε την κίνηση του τροχού από το ανώτερο σημείο της στεφάνης (όπου έχει $υ_{op} = 2m/s$), μέχρι το σημείο που το κέντρο μάζας του απέχει απόσταση R από το οριζόντιο δάπεδο.

<u>Α.Δ.Μ.Ε</u>: $E_{MHX}^{αρχ} = E_{MHX}^{τελ} \Rightarrow K_{αρχ} + U_{αρχ} = K_{τελ} + U_{τελ} \Rightarrow$

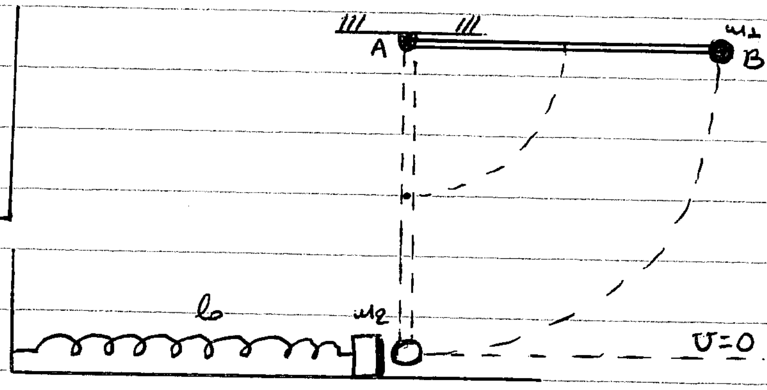
$\Rightarrow K_{min} + m_1 g 2(R-r_1) = \frac{1}{2} m_1 υ^2 + \frac{1}{2} I ω^2 + m_1 g (R-r_1) \Rightarrow$

$\Rightarrow K_{\min} + m_1 g(R - r_1) = \frac{3}{4} m_1 \upsilon^2 \Rightarrow \upsilon = \sqrt{\frac{28}{3}}\ m/s$

• Στο σημείο της τροχιάς που η απόσταση του κέντρου μάζας του τροχού απέχει απόσταση R από το έδαφος ισχύει:

$F_K = m_1 \frac{\upsilon^2}{R - r_1} \xrightarrow{F_K = \Sigma F_x} N = m_1 \frac{\upsilon^2}{R - r_1} \Rightarrow \boxed{N = 140N}$

3.18 $L = 0,5m$, $M = 3kg$
$m_1 = 1kg$, $\omega_0 = 5\sqrt{23}\ \frac{rad}{s}$
$K = 500\frac{N}{m}$, $m_2 = 5kg$



A. • Α.Δ.Μ.Ε

$E_{MHX}^{\alpha\rho\chi} = E_{MHX}^{\tau\varepsilon\lambda} \Rightarrow K_{\alpha\rho\chi} + U_{\alpha\rho\chi} = K_{\tau\varepsilon\lambda} + U_{\tau\varepsilon\lambda} \Rightarrow \frac{1}{2} I_A \omega_0^2 + MgL + m_1 g L =$
$= \frac{1}{2} I_A \omega^2 + Mg\frac{L}{2} \Rightarrow \frac{1}{2} I_A \omega^2 = \frac{1}{2} I_A \omega_0^2 + Mg\frac{L}{2} + m_1 g L$ ①

• $I_A = I_A^{\rho\alpha\beta\delta o\upsilon} + I_A^{m_1}$

Θεώρημα Steiner: $I_A^{\rho\alpha\beta\delta o\upsilon} = I_{cm} + M\frac{L^2}{4} = \frac{1}{12}ML^2 + M\frac{L^2}{4}$ ή $I_A = \frac{ML^2}{3}$ $\Rightarrow$

$\Rightarrow I_A = \frac{1}{3}ML^2 + m_1 L^2 \Rightarrow \underline{I_A = 0,5\ kgm^2}$

① $\overset{S.I.}{\Rightarrow} \frac{1}{2} \cdot 0,5\,\omega^2 = \frac{1}{2} \cdot 0,5 \cdot 575 + 7,5 + 5 \Rightarrow \omega^2 = 575 + 50$
$\Rightarrow \omega = \sqrt{625} \Rightarrow \underline{\omega = 25\ rad/s}$

Άρα $\upsilon_1 = \omega L \Rightarrow \boxed{\upsilon_1 = 12,5\ m/s}$

B. • Α.Δ. Στροφορμής: $\vec{L}_{\alpha\rho\chi} = \vec{L}_{\tau\varepsilon\lambda}$

$I_A \omega = m_2 \upsilon L - I_A \omega' \xrightarrow{\omega' = \frac{\omega}{5} = 5\,rad/s}$

$0,5 \cdot 25 = 5 \cdot \upsilon \cdot 0,5 - 0,5 \cdot 5 \Rightarrow \boxed{\upsilon = 6\ m/s}$

Γ. • Η κρούση έγινε στη θέση ισορροπίας του m_2, άρα:

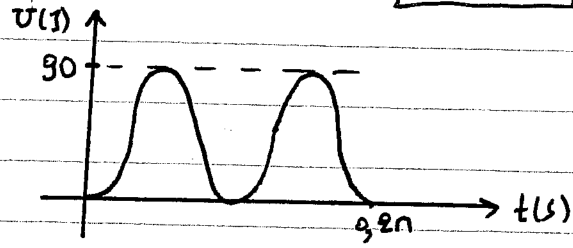
$\upsilon = \upsilon_{max} = \omega A \xrightarrow{\omega = \sqrt{\frac{K}{m_2}} = 10\,rad/s} \underline{A = 0,6m}$

$x = A\eta\mu(\omega t+\varphi_0) \xrightarrow[x=0]{t_0=0} 0 = A\eta\mu\varphi_0 \Rightarrow \eta\mu\varphi_0 = 0 = \eta\mu 0 \Rightarrow \begin{cases}\varphi_0 = 2k\pi+0 \\ \varphi_0 = 2k\pi+\pi-0\end{cases} \xRightarrow{k=0}$

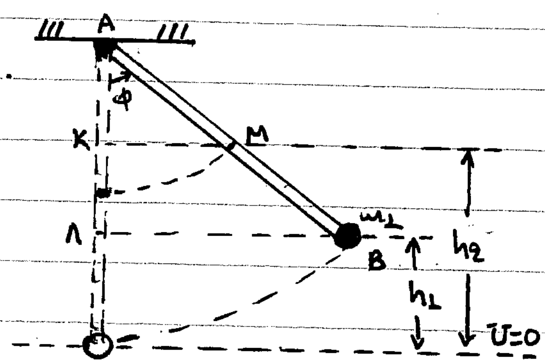
$\Rightarrow \begin{cases} \varphi_0 = 0, & \upsilon>0 \\ \varphi_0=\pi, & \upsilon<0\end{cases}$ Άρα $\varphi_0 = \pi$ rad

• $x = 0{,}6\eta\mu(10t+\pi)$, (S.I.)

• $U = \frac{1}{2} Dx^2 \xrightarrow{D=k} \boxed{U = 90\eta\mu^2(10t+\pi)}$, (S.I.)



Δ.



• Α.Δ.Μ.Ε.: $E_{\mu\eta\chi}^{\alpha\rho\chi} = E_{\mu\eta\chi}^{\tau\varepsilon\lambda} \Rightarrow$

$K_{\alpha\rho\chi} + U_{\alpha\rho\chi} = K_{\tau\varepsilon\lambda} + U_{\tau\varepsilon\lambda} \Rightarrow$

$\frac{1}{2} I_A \omega'^2 + Mg\frac{L}{2} = m_1 g h_1 + Mg h_2$ (2)

• $A\hat{\Lambda}B$: $\sigma\upsilon\nu\varphi = \frac{(A\Lambda)}{(AB)} \Rightarrow (A\Lambda) = L\,\sigma\upsilon\nu\varphi$, $h_1 = L - L\,\sigma\upsilon\nu\varphi$

• $A\hat{K}M$: $\sigma\upsilon\nu\varphi = \frac{(AK)}{(AM)} \Rightarrow (AK) = \frac{L}{2}\sigma\upsilon\nu\varphi$, $h_2 = L - \frac{L}{2}\sigma\upsilon\nu\varphi$

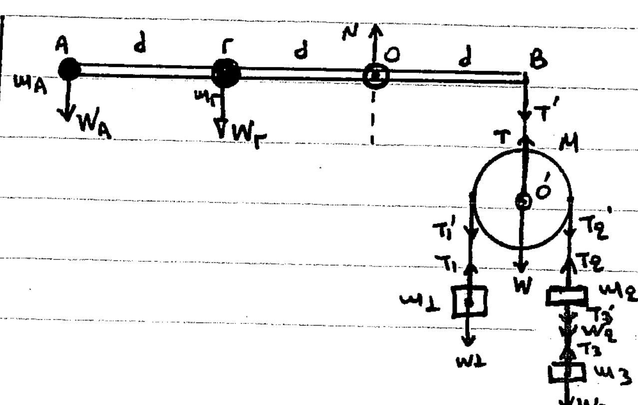
(2) $\Rightarrow \frac{1}{2}\cdot 0{,}5\cdot 25 + 7{,}5 = 10(0{,}5 - 0{,}5\,\sigma\upsilon\nu\varphi) + 30\left(0{,}5 - \frac{0{,}5}{2}\sigma\upsilon\nu\varphi\right)$

$\Rightarrow 6{,}25 + 7{,}5 = 5 - 5\sigma\upsilon\nu\varphi + 15 - 7{,}5\sigma\upsilon\nu\varphi \Rightarrow$

$\Rightarrow 12{,}5\,\sigma\upsilon\nu\varphi = 20 - 13{,}75 \Rightarrow \sigma\upsilon\nu\varphi = \frac{6{,}25}{12{,}5} \Rightarrow$

$\Rightarrow \sigma\upsilon\nu\varphi = \frac{1}{2}$ ή $\boxed{\varphi = \frac{\pi}{3}\text{ rad}}$

3.19 Αβαρής ράβδος, 3d (d=1m)
(ΑΟ)=2d, $m_A = 1$ kg
(ΟΓ)=d, $m_\Gamma = 6$ kg
(ΟΒ)=d, M=4kg, $m_1 = 2$ kg
$m_2 = m_3 = 1$ kg



Α. • Για το σύστημα τροχαλίας - μάζες m_1, m_2, m_3 ως προς τον άξονα O', ισχύει:

■ $\Sigma\tau_{(O')} = W_1 \cdot R - W_{2,3} \cdot R = m_1 g R - (m_2+m_3) g R \Rightarrow$

$\Rightarrow \Sigma\tau_{(O')} = (m_1 - m_2 - m_3) \cdot g \cdot R \Rightarrow \underline{\Sigma\tau_{(O')} = 0}$

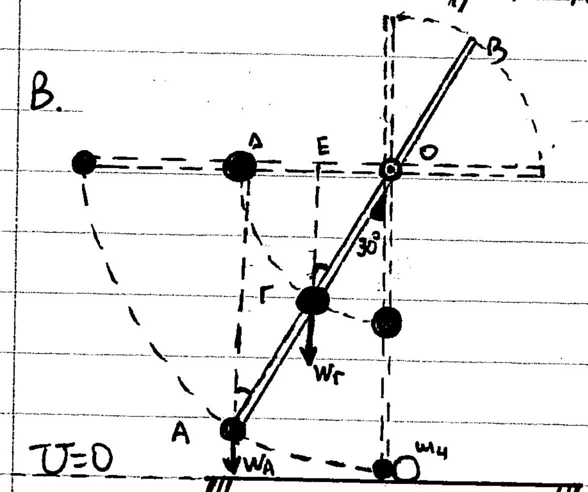
δηλαδή το σύστημα ισορροπεί από στροφική σκοπιά.

• Για το σύστημα ράβδου, μαζών m_A, m_Γ, τροχαλίας, μάζες m_1, m_2, m_3 ως προς τον άξονα O, ισχύει:

■ $\Sigma\tau_{(O)} = W_A \cdot 2d + W_\Gamma \cdot d - W_{\tau\rho,1,2,3} \cdot d = (2m_A + m_\Gamma - M - m_1 - m_2 - m_3) g \cdot d \Rightarrow \underline{\Sigma\tau_{(O)} = 0}$

Άρα το σύστημα ισορροπεί με τη ράβδο σε οριζόντια θέση

Β.



$\Sigma\tau = I_O \cdot \alpha_{\gamma\omega\nu} \Rightarrow W_A \cdot (O\Delta) + W_\Gamma \cdot (OE) =$
$= I_O \cdot \alpha_{\gamma\omega\nu} \Rightarrow m_A g (O\Delta) + m_\Gamma g (OE) = I_O \cdot \alpha_{\gamma\omega\nu}$ (1)

• $O\hat{\Delta}A$: ημ30° $= \frac{(O\Delta)}{(OA)} \Rightarrow (O\Delta) = 2d \cdot \frac{1}{2} = d$

• $O\hat{E}\Gamma$: ημ30° $= \frac{(OE)}{(O\Gamma)} \Rightarrow (OE) = \frac{d}{2}$

(1) $\Rightarrow m_A \cdot g \cdot d + m_\Gamma g \frac{d}{2} = [m_A (2d)^2 + m_\Gamma \cdot d^2] \cdot \alpha_{\gamma\omega\nu} \Rightarrow$

$\overset{S.I.}{\Rightarrow} 10 + 30 = (4+6) \cdot \alpha_{\gamma\omega\nu} \Rightarrow \boxed{\alpha_{\gamma\omega\nu} = 4\ rad/s^2}$

Γ. • Α.Δ.Μ.Ε.: $E_{MHX}^{\alpha\rho\chi} = E_{MHX}^{\tau\epsilon\lambda} \Rightarrow K_{\alpha\rho\chi} + U_{\alpha\rho\chi} = K_{\tau\epsilon\lambda} + U_{\tau\epsilon\lambda} \Rightarrow$

$\Rightarrow m_A g \cdot 2d + m_\Gamma g \cdot 2d = \frac{1}{2} I_O \omega^2 + m_\Gamma g d \xrightarrow{I_O = (m_A 4d^2 + m_\Gamma d^2) = 10\ kgm^2}$

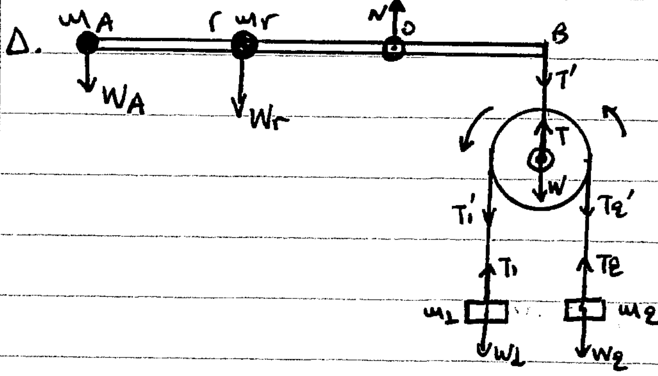
$\overset{S.I.}{\Rightarrow} 20 + 120 = 5\omega^2 + 60 \Rightarrow \omega^2 = 16 \Rightarrow \underline{\omega = 4\ rad/s}$

• Α.Δ. Στροφορμής: $L_{\alpha\rho\chi} = L_{\tau\epsilon\lambda}$

$I_O \cdot \omega = [I_O + m_4 (2d)^2]\, \omega' \Rightarrow \underline{\omega' = \frac{4}{3}\ rad/s}$

■ $U_A = \omega' \cdot 2d \Rightarrow \boxed{U_A = \frac{8}{3}\ m/s}$

Δ.



• Σώμα m_1: $\Sigma F = m_1 a_{cm} \Rightarrow$

$\Rightarrow W_1 - T_1 = m_1 a_{cm} \Rightarrow T_1 = m_1 g - m_1 a_{cm}$ (1)

• Σώμα m_2: $\Sigma F = m_2 a_{cm} \Rightarrow$

$\Rightarrow T_2 - W_2 = m_2 a_{cm} \Rightarrow T_2 = m_2 g + m_2 a_{cm}$ (2)

• Τροχαλία: $\Sigma \tau = I a_{γων} \xrightarrow{a_{γων} = \frac{a_{cm}}{R}}$

$$\Rightarrow T_1' R - T_2' R = \frac{1}{2} M R^2 \frac{a_{cm}}{R} \xrightarrow[T_2' = T_2]{T_1' = T_1} T_1 - T_2 = \frac{1}{2} M a_{cm} \xrightarrow{(1),(2)}$$

$$\Rightarrow m_1 g - m_1 a_{cm} - m_2 g - m_2 a_{cm} = \frac{1}{2} M a_{cm} \Rightarrow \underline{a_{cm} = 2\, m/s^2}$$

$(1) \Rightarrow T_1 = 16N \; (= T_1')$

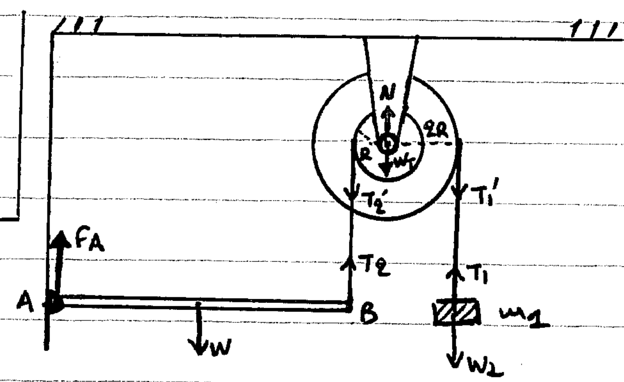
$(2) \Rightarrow T_2 = 12N \; (= T_2')$

$$\Sigma F = 0 \Rightarrow T - W - T_1' - T_2' = 0 \Rightarrow \underline{T = 68N \; (= T')}$$

• Ράβδος: $\Sigma \tau_{(O)} = 0 \Rightarrow W_A \cdot 2d + W_r \cdot d - T' d = 0 \Rightarrow$

$$\Rightarrow m g \cdot 2d + m_r g d - T' d = 0 \Rightarrow m = \frac{T' - m_r g}{2g} \Rightarrow \boxed{m = 0{,}4\, kg}$$

3.20 $L = 0{,}6m$, $M = 2\, kg$
$R = 0{,}1m$, $m = 6\, kg$
$I_{cm} = \frac{1}{12} M L^2$



A. Το σύστημα ισορροπεί:

• Σώμα m_1: $\Sigma F_y = 0 \Rightarrow T_1 - W_1 = 0 \Rightarrow T_1 = m_1 g \Rightarrow m_1 = \frac{T_1}{g}$ (1)

• Ράβδος: $\Sigma \tau_{(A)} = 0 \Rightarrow -W \cdot \frac{L}{2} + T_2 \cdot L = 0 \Rightarrow T_2 = \frac{Mg}{2} \Rightarrow \underline{T_2 = 10N (= T_2')}$

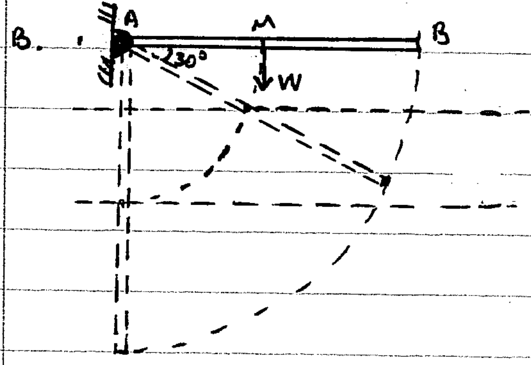
$\Sigma F_y = 0 \Rightarrow F_A + T_2 - W = 0 \Rightarrow \boxed{F_A = 10N}$

• Τροχαλία: $\Sigma \tau = 0 \Rightarrow T_2' R - T_1' 2R = 0 \Rightarrow T_1' = \frac{T_2'}{2} \Rightarrow \underline{T_1' = 5N (= T_1)}$

$\Sigma F_y = 0 \Rightarrow N - W_T - T_2' - T_1' = 0 \Rightarrow \boxed{N = 75N}$

Από $(1) \Rightarrow \boxed{m_1 = 0{,}5\, kg}$

Β.



1) $\frac{\Delta L}{\Delta t} = \Sigma \tau = w \cdot \frac{L}{2}$ ή $\boxed{\frac{dL}{dt} = 6\, kg m^2/s^2}$

2) Επιλέγω ως επίπεδο μηδενικής δυναμικής ενέργειας τη νέα θέση του κέντρου μάζας της ράβδου.

ΑΔΜΕ: $E_{MHX}^{αρχ} = E_{MHX}^{τελ} \Rightarrow K_{αρχ} + U_{αρχ} = K_{τελ} + U_{τελ}$

$\Rightarrow Mg \frac{L}{2} ημ30 = \frac{1}{2} I_A \omega^2$ ②

Θεώρημα Steiner: $I_A = I_{cm} + M\frac{L^2}{4} \Rightarrow I_A = \frac{1}{12}ML^2 + \frac{1}{4}ML^2 \Rightarrow$

$\Rightarrow \underline{I_A = \frac{1}{3}ML^2}$

② $\Rightarrow Mg \cdot \frac{L}{2} \cdot \frac{1}{2} = \frac{1}{2} \cdot \frac{1}{3} M L^2 \omega^2 \Rightarrow \frac{g}{2} = \frac{L}{3}\omega^2 \Rightarrow \underline{\omega = 5\, rad/s}$

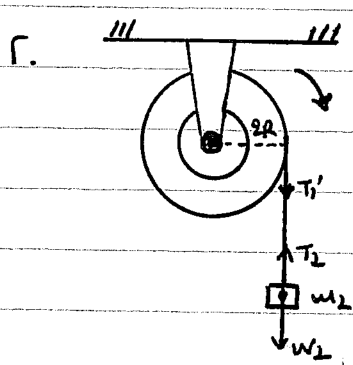
Άρα $L = I_A \cdot \omega \Rightarrow L = \frac{1}{3}ML^2\omega \Rightarrow \boxed{L = 1,2\, kg m^2/s}$

3) Επιλέγω ως επίπεδο μηδενικής δυναμικής ενέργειας τη νέα θέση του κέντρου μάζας της ράβδου.

Α.ΔΜ.Ε.: $E_{MHX}^{αρχ} = E_{MHX}^{τελ} \Rightarrow K_{αρχ} + U_{αρχ} = K_{τελ} + U_{τελ} \Rightarrow$

$Mg\frac{L}{2} = K \Rightarrow \boxed{K = 6\, J}$

Γ.



1) • Σώμα m_2: $\Sigma F_y = m_1 \alpha_{cm} \Rightarrow W_1 - T_1 = m_1 \cdot \alpha_{cm} \Rightarrow$

$\Rightarrow T_1 = m_1 g - m_1 \alpha_{cm} \xrightarrow{\alpha_{cm} = \alpha_{γων} 2R = 1\, m/s^2}$

$\Rightarrow \underline{T_1 = 4,5\, N\ (= T_1')}$

• Τροχαλία: $\Sigma \tau = I \cdot \alpha_{γων} \Rightarrow T_1' \, 2R = I \cdot \alpha_{γων} \Rightarrow$

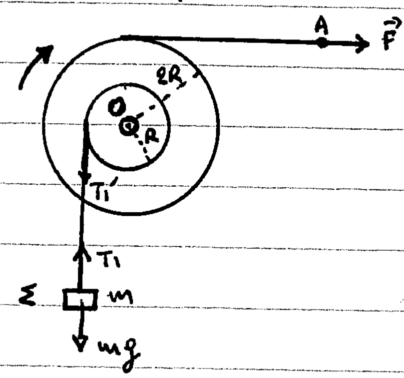
$\xrightarrow{T_1' = T_1} \boxed{I = 0,18\, kg \cdot m^2}$

2) $\frac{dL}{dt} = \Sigma \tau = w_2 \cdot 2R = m_1 g\, 2R$ ή $\boxed{\frac{dL}{dt} = 1\, kg m^2/s^2}$

3) • $P_{T_1'} = \tau_{T_1'} \cdot \omega \xrightarrow{\omega = \alpha_{γων} t_1 = 10\, rad/s} P_{T_1'} = T_1' \cdot 2R \cdot \omega \Rightarrow \boxed{P_{T_1'} = 9\, W}$

• $L_{m_2} = m_2 v_{cm} 2R \xrightarrow{v_{cm} = \alpha_{cm} t_1 = 2\, m/s} \boxed{L_{m_2} = 0,2\, kg m^2/s}$

3.21 $M=10kg$, $R=0,2m$, $I=MR^2$, $m=20kg$



A. Το σύστημα ισορροπεί

$\Sigma\tau_{(O)}=0 \Rightarrow mgR-F\cdot 2R=0 \Rightarrow$

$\Rightarrow F=\frac{mg}{2} \Rightarrow \boxed{F=100N}$

B. · Σώμα m: $\Sigma F=m\alpha_{cm} \Rightarrow T_1-mg=m\alpha_{cm} \Rightarrow T_1=mg+m\alpha_{cm}$ (1)

· Τροχαλία: $\Sigma\tau=I\cdot\alpha_{\gamma\omega\nu} \xrightarrow{\alpha_{\gamma\omega\nu}=\alpha_{cm}/R} F\cdot 2R-T_1'R=MR^2\frac{\alpha_{cm}}{R} \Rightarrow$

$\xRightarrow{(1)} 2F-mg-m\alpha_{cm}=M\alpha_{cm} \Rightarrow 2F-mg=\alpha_{cm}(M+m) \Rightarrow \boxed{\alpha_{cm}=1\frac{m}{s^2}}$

Γ. · $h=\frac{1}{2}\alpha_{cm}t_1^2 \Rightarrow \underline{t_1=2s}$

· $\omega=\alpha_{\gamma\omega\nu}t_1 \xrightarrow{\alpha_{\gamma\omega\nu}=\frac{\alpha_{cm}}{R}=5rad/s^2} \underline{\omega=10rad/s}$

Άρα $L=I\omega \Rightarrow L=MR^2\omega \Rightarrow \boxed{L=4kgm^2/s}$

Δ. · $\theta=\frac{1}{2}\alpha_{\gamma\omega\nu}t_1^2 \Rightarrow \underline{\theta=10rad}$

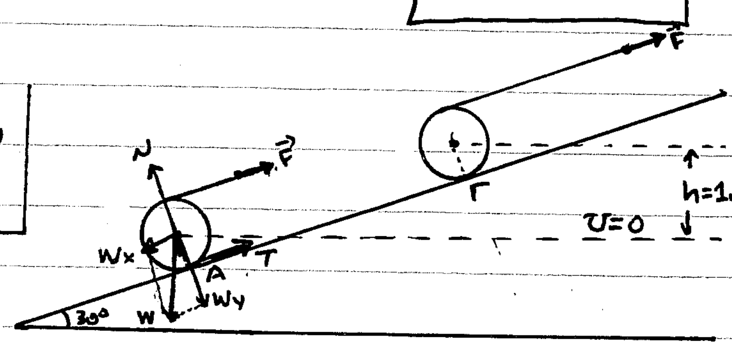
· $\theta=\frac{s}{2R} \Rightarrow S=2R\theta \Rightarrow S=4m$

(2ος τρόπος: $\alpha_A=\alpha_{\gamma\omega\nu}\cdot 2R=2m/s^2$, $S=\frac{1}{2}\alpha_A t_1^2=4m$)

Ε. $\Pi\%=\frac{K_\pi}{W_F}\cdot 100\%=\frac{\frac{1}{2}I\omega^2}{F\cdot S}100\%=\frac{\frac{1}{2}MR^2\omega^2}{F\cdot S}\cdot 100\%$

$\Rightarrow \Pi\%=\frac{20}{460}\cdot 100\% \Rightarrow \boxed{\Pi\%=\frac{100}{23}\%}$

3.22 $M=40kg$, $R=0,2m$, $\varphi=30°$



A. Ο κύλινδρος ανεβαίνει με σταθερή ταχύτητα:

$\cdot \Sigma\tau=0 \Rightarrow F\cdot R-T\cdot R=0 \Rightarrow \underline{T=F}$ (1)

$\cdot \Sigma F_x=0 \Rightarrow F+T-W_x=0 \xRightarrow{(1)} 2F=Mg\eta\mu 30° \Rightarrow \boxed{F=100N}$

B. Ο κύλινδρος κυλίεται χωρίς να ολισθαίνει με $\upsilon_0=0$, από το Α:

• $\Sigma\tau = I\alpha_{\gamma\omega\nu} \xrightarrow{\alpha_{\gamma\omega\nu}=\alpha_{cm}/R} F\cdot R - T\cdot R = \frac{1}{2}MR^2\frac{\alpha_{cm}}{R} \Rightarrow T = F - \frac{M}{2}\alpha_{cm}$ (1)

• $\Sigma F_x = M\alpha_{cm} \Rightarrow F + T - w_x = M\alpha_{cm} \xrightarrow{(1)} F + F - \frac{M}{2}\alpha_{cm} - Mg\eta\mu 30^\circ =$
$= M\alpha_{cm} \Rightarrow 2F - Mg\eta\mu 30^\circ = \frac{3}{2}M\alpha_{cm} \Rightarrow \boxed{\alpha_{cm} = 1\, m/s^2}$

Γ. • $\eta\mu 30 = \frac{h}{s} \Rightarrow s = 2m$

• $s = \frac{1}{2}\alpha_{cm}t^2 \Rightarrow t = 2s$

• $\alpha_{\gamma\omega\nu} = \alpha_{cm}/R \Rightarrow \alpha_{\gamma\omega\nu} = 5\, rad/s^2$

• $\omega = \alpha_{\gamma\omega\nu} t \Rightarrow \omega = 10\, rad/s$

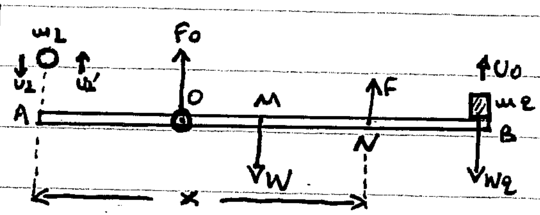
Άρα $L = I\omega \Rightarrow L = \frac{1}{2}MR^2\omega \Rightarrow \boxed{L = 8\, kgm^2/s}$

Δ. • $W_F = F\cdot s + F\cdot R\cdot\theta \xrightarrow{\theta = \frac{1}{2}\alpha_{\gamma\omega\nu}t^2 = 10\, rad} W_F = 260J + 260J \Rightarrow$
$\Rightarrow \boxed{W_F = 520J}$

• $\Delta E_{\mu\eta\chi} = E_{\mu\eta\chi}^{\tau\varepsilon\lambda} - E_{\mu\eta\chi}^{\alpha\rho\chi} = K_{\tau\varepsilon\lambda} + U_{\tau\varepsilon\lambda} - (K_{\alpha\rho\chi} + U_{\alpha\rho\chi}) \Rightarrow$
$\Rightarrow \Delta E_{\mu\eta\chi} = \frac{1}{2}MU_{cm}^2 + \frac{1}{2}I\omega^2 + Mgh - 0 \Rightarrow \Delta E_{\mu\eta\chi} = \frac{1}{2}MU_{cm}^2 + \frac{1}{2}\frac{1}{2}MR^2\omega^2 +$
$+ Mgh \Rightarrow \Delta E_{\mu\eta\chi} = \frac{3}{4}MU_{cm}^2 + Mgh \xrightarrow{U_{cm} = \alpha_{cm}t = 2m/s} \boxed{\Delta E_{\mu\eta\chi} = 520J}$

Άρα $\boxed{W_F = \Delta E_{\mu\eta\chi}\ (=520J)}$

3.23. $m_1 = 2kg$, $U_1 = 12\frac{m}{s}$
$M = 16kg$, $L = 0,3m$
$(AO) = \frac{L}{3}$
$m_2 = 1kg$, $F = 100N$
$U_1' = 8\, m/s$, U_0
$h = 0,2m$, $m_3 = 4kg$, $k = 125\frac{N}{m}$



Α. Το σύστημα ράβδος - σώμα m_2 ισορροπεί:

• $\Sigma\tau_{(O)} = 0 \Rightarrow F\cdot(ON) - W\cdot(OM) - W_2(OB) = 0$
$\xrightarrow[(OB) = L - \frac{L}{3} = 2\frac{L}{3}]{(OM) = \frac{L}{2} - \frac{L}{3} = \frac{L}{6}} (ON) = 0,1m$

Άρα $(AN) = (AO) + (ON) \xrightarrow{(AO) = \frac{L}{3} = 0,1m} (AN) = 0,2m \xrightarrow{(AN)=x} \boxed{x = 0,2m}$

• $\Sigma F_y = 0 \Rightarrow F_0 + F - W - W_2 = 0 \Rightarrow \boxed{F_0 = 70N}$

Β. • Α.Δ. Στροφορμής: $\vec{L}_{\alpha\rho\chi} = \vec{L}_{\tau\varepsilon\lambda}$

$m_1 u_1 \cdot (AO) = [I_O + m_2 (OB)^2]\cdot \omega - m_1 u_1' (AO)$ (1)

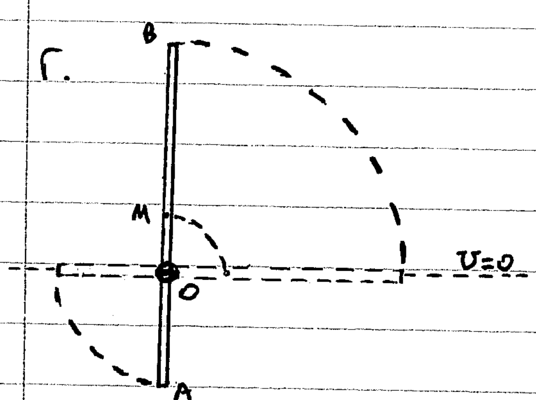
Θεώρημα Steiner: $I_O = I_{cm} + M\frac{L^2}{36} = \frac{1}{12}ML^2 + \frac{1}{36}ML^2$ ή $\underline{I_O = \frac{ML^2}{9}}$

(1) $\xRightarrow{S.I.}$ $2\cdot 12\cdot \frac{0,3}{3} = \left(16\cdot\frac{0,09}{9} + 1\cdot 0,04\right)\cdot\omega - 2\cdot 8\cdot 0,1$ ⇒

⇒ $2,4 + 1,6 = 0,2\,\omega$ ⇒ $\underline{\omega = 20\, rad/s}$

Άρα $u_0 = \omega\cdot(OB) \xRightarrow{(OB)=2L/3} \boxed{u_0 = 4\, m/s}$

Γ.



Θ.Μ.Κ.Ε.: $\Sigma W_F = \Delta K$ ⇒

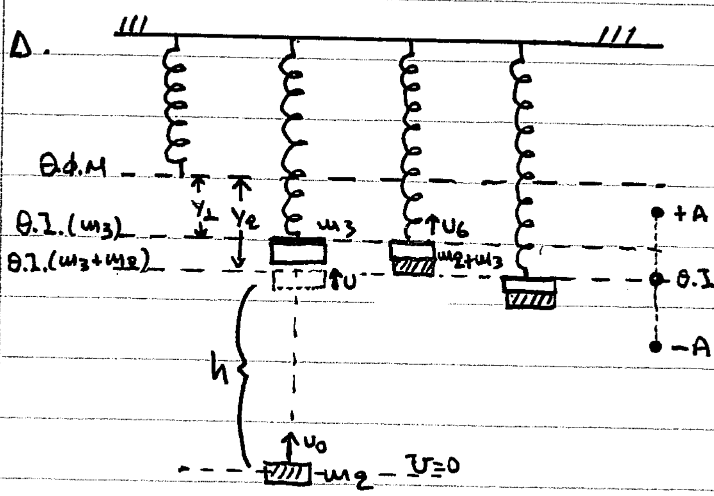
$W_{F\varepsilon\xi} + W_w = K_{\tau\varepsilon\lambda} - K_{\alpha\rho\chi} \xRightarrow{W_w = -\Delta U}$

$W_{F\varepsilon\xi} + U_{\alpha\rho\chi} - U_{\tau\varepsilon\lambda} = -\frac{1}{2} I_O \omega^2$ ⇒

$W_{F\varepsilon\xi} - Mg\frac{L}{6} = -\frac{1}{2}\cdot\frac{ML^2}{9}\,\omega^2$ ⇒

⇒ $\boxed{W_{F\varepsilon\xi} = -24\, J}$

Δ.



• <u>Κίνηση m2:</u>

ΑΔΜΕ: $E_{MHX}^{\alpha\rho\chi} = E_{MHX}^{\tau\varepsilon\lambda}$ ⇒

$K_{\alpha\rho\chi} + U_{\alpha\rho\chi} = K_{\tau\varepsilon\lambda} + U_{\tau\varepsilon\lambda}$ ⇒

$\frac{1}{2} m_2 u_0^2 = m_2 g h + \frac{1}{2} m_2 u^2$ ⇒

⇒ $\underline{u = 2\sqrt{3}\, m/s}$

• κρούση ανελαστική (πλαστική)

Α.Δ.Ορμής: $\vec{p}_{\alpha\rho\chi} = \vec{p}_{\tau\varepsilon\lambda}$

$m_2 u = (m_2 + m_3) u_6$ ⇒

⇒ $\underline{u_6 = 0,4\sqrt{3}\, m/s}$

• Θ.Ι.(m3): $\Sigma F_y = 0 \Rightarrow F_{\varepsilon\lambda} - w_3 = 0 \Rightarrow k y_1 = m_3 g \Rightarrow \underline{y_1 = 0,32 m}$

• Θ.Ι.(m2+m3): $\Sigma F_y = 0 \Rightarrow F'_{\varepsilon\lambda} - w_{2,3} = 0 \Rightarrow k\cdot y_2 = (m_2+m_3) g \Rightarrow \underline{y_3 = 0,40 m}$

• Α.Δ.Ε. Ταλάντωσης: $E = K + U \Rightarrow \frac{1}{2}kA^2 = \frac{1}{2}(m_2+m_3)u_6^2 + \frac{1}{2}k(y_2 - y_1)^2$ ⇒

⇒ $\boxed{A = 0,16 m}$

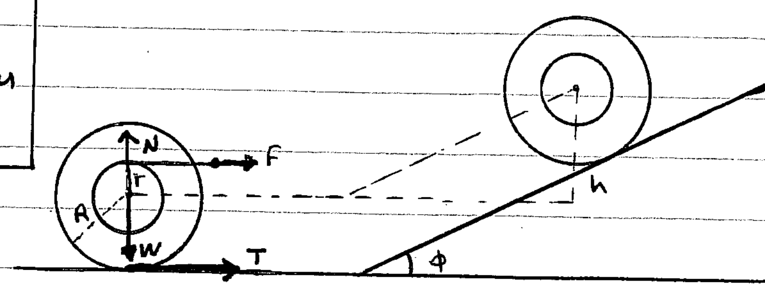
• $k=(m_2+m_3)\omega^2 \Rightarrow \omega=\sqrt{\frac{k}{m_2+m_3}} \Rightarrow \underline{\omega=5\,rad/s}$

• $y=A\eta\mu(\omega t+\varphi_0) \xrightarrow[y=y_2-y_1]{t_0=0} 0{,}08=0{,}16\,\eta\mu\varphi_0 \Rightarrow \eta\mu\varphi_0=\frac{1}{2}=\eta\mu\frac{\pi}{6} \Rightarrow$

$\Rightarrow \begin{cases}\varphi_0=2k\pi+\frac{\pi}{6}\\ \varphi_0=2k\pi+\pi-\frac{\pi}{6}\end{cases} \xrightarrow{k=0} \begin{cases}\varphi_0=\frac{\pi}{6}, & \upsilon>0\\ \varphi_0=\frac{5\pi}{6}, & \upsilon<0\end{cases}$ ή $\underline{\varphi_0=\frac{\pi}{6}\,rad}$

Άρα $\upsilon=\upsilon_{max}\sigma\upsilon\nu(\omega t+\varphi_0) \xrightarrow{\upsilon_{max}=\omega A} \boxed{\upsilon=0{,}8\sigma\upsilon\nu(5t+\frac{\pi}{6})}$, (S.I.)

3.24 $m_k=2kg$, $r=0{,}1m$
$M=0{,}5kg$, $R=0{,}2m$
$F=10N$, $\upsilon_0=0$



A. $I=I_k+2I_\delta \Rightarrow I=\frac{1}{2}m_k r^2+2\cdot\frac{1}{2}MR^2 \Rightarrow \boxed{I=0{,}03\,kgm^2}$

B. Το σύστημα κυλίεται χωρίς να ολισθαίνει:

• $\Sigma\tau=I\alpha_{\gamma\omega\nu} \xrightarrow{\alpha_{\gamma\omega\nu}=\alpha_{cm}/R} F\cdot r-T\cdot R=I\cdot\frac{\alpha_{cm}}{R} \xrightarrow{S.I.} T=5-\frac{3}{4}\alpha_{cm}$ (1)

• $\Sigma F=m_{o\lambda}\cdot\alpha_{cm} \Rightarrow F+T=(m_k+2M)\cdot\alpha_{cm} \xrightarrow[S.I.]{(1)} 10+5-0{,}75\cdot\alpha_{cm}=3\cdot\alpha_{cm}$

$\Rightarrow \underline{\alpha_{cm}=4m/s^2}$

► $\frac{dL}{dt}=\Sigma\tau=I\cdot\alpha_{\gamma\omega\nu} \xrightarrow{\alpha_{\gamma\omega\nu}=\frac{\alpha_{cm}}{R}=20rad/s^2} \boxed{\frac{dL}{dt}=0{,}6\,kgm^2/s^2}$

Γ. • $P_F=F\cdot\upsilon_{cm}+\tau_F\cdot\omega=F\cdot\upsilon_{cm}+F\cdot r\cdot\omega$ (2)

$\upsilon_{cm}=\alpha_{cm}t_1=4\cdot t_1$

$\omega=\alpha_{\gamma\omega\nu}t_1=20\cdot t_1$

Άρα $(2) \xrightarrow{S.I.} 60=10\cdot4t_1+10\cdot0{,}1\cdot20t_1 \Rightarrow 60=60t_1 \Rightarrow \underline{t_1=1s}$

• $\theta=\frac{1}{2}\alpha_{\gamma\omega\nu}t_1^2 \Rightarrow \theta=10\,rad$

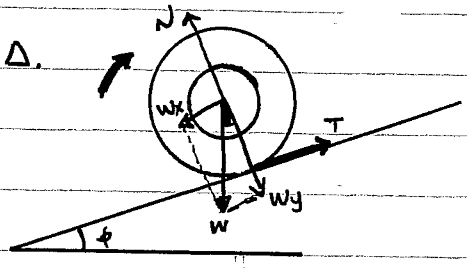
$\theta=\frac{s}{r} \xrightarrow{s=\ell} \ell=\theta\cdot r \Rightarrow \boxed{\ell=1m}$

• $W_F=F\cdot x_{cm}+\tau_F\cdot\theta=F\cdot x_{cm}+F\cdot r\cdot\theta$ (3)

$x_{cm}=\frac{1}{2}\alpha_{cm}t_1^2 \Rightarrow x_{cm}=2m$

(3) $\Rightarrow W_F = (10 \cdot 2 + 20 \cdot 0,1 \cdot 10)\,J \Rightarrow \boxed{W_F = 30\,J}$

Δ.



• $\Sigma\tau = I \cdot \alpha'_{\gamma\omega\nu} \xrightarrow{\alpha'_{\gamma\omega\nu} = \alpha'_{cm}/R} -TR = I\frac{\alpha'_{cm}}{R} \Rightarrow$

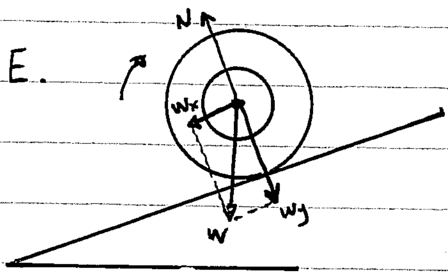
$\Rightarrow T = -\frac{I\alpha'_{cm}}{R^2}$ (4)

• $\Sigma F = m_{o\lambda} \cdot \alpha'_{cm} \Rightarrow T - W_x = m_{o\lambda}\,\alpha'_{cm} \overset{(4)}{\Rightarrow}$

$-\frac{I\alpha'_{cm}}{R^2} - m_{o\lambda} g\,\eta\mu\phi = m_{o\lambda}\alpha'_{cm} \Rightarrow$

$\Rightarrow \alpha'_{cm} = -4\,m/s^2$ ή $\boxed{|\alpha'_{cm}| = 4\,m/s^2}$

E.



• $\upsilon_{cm} = \alpha_{cm} t_1 \xrightarrow{t_1 = 1s} \upsilon_{cm} = 4\,m/s$ (αρχική ταχύτητα με την οποία ανέρχεται το κεκλιμένο επίπεδο).

• $\upsilon'_{cm} = \upsilon_0 - |\alpha'_{cm}|\,t \xrightarrow{t = 0,5s} \upsilon'_{cm} = (4 - 4 \cdot 0,5)$

$\Rightarrow \upsilon'_{cm} = 2\,m/s$ (αρχική ταχύτητα με την οποία συναντά το λείο κεκλιμένο επίπεδο).

• Στροφική κίνηση: Ομαλή κυκλική ($\Sigma\tau = 0$)

• Μεταφορική κίνηση: $\Sigma F = m_{o\lambda}\alpha''_{cm} \Rightarrow -W_x = m_{o\lambda} \cdot \alpha''_{cm} \Rightarrow -m_{o\lambda} g\,\eta\mu\phi = m_{o\lambda}\alpha''_{cm}$

$\Rightarrow \underline{\alpha''_{cm} = -g\,\eta\mu\phi = -5\,m/s^2}$

$t = \frac{\upsilon'_{cm}}{|\alpha''_{cm}|} = 0,4\,s$ μέχρι να σταματήσει

• 0 - 1s : $L = I\omega = I\alpha_{\gamma\omega\nu}t \Rightarrow \underline{L = 0,6t} \xrightarrow{t=0} L = 0$

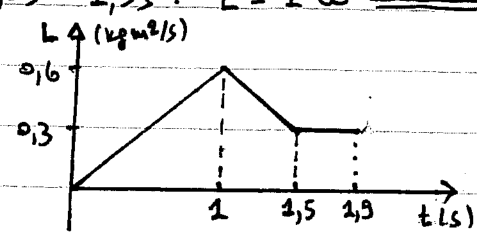
$\xrightarrow{t=1s} L = 0,6\,kgm^2/s$

• 1s - 1,5s : $L = I\omega = I(\omega_0 - \alpha'_{\gamma\omega\nu}\Delta t) \xrightarrow[|\alpha'_{\gamma\omega\nu}| = |\alpha'_{cm}/R| = 20\,rad/s^2]{\omega_0 = \upsilon_{cm}/R = 20\,rad/s} L = 0,03\,[20 - 20(t-1)]$

$\Rightarrow \underline{L = 1,2 - 0,6t} \xrightarrow{t=1s} L = 0,6\,kgm^2/s$

$\xrightarrow{t=1,5s} L = 0,3\,kgm^2/s$

• 1,5s - 1,9s : $L = I\omega \xrightarrow{\omega = \upsilon'_{cm}/R = 10\,rad/s} \underline{L = 0,3\,kgm^2/s}$ (σταθερή).



ΚΕΦΑΛΑΙΟ 3ο – ΘΕΜΑ 4ο

3.25

$M = 2\,kg$, $R = 0{,}2\,m$

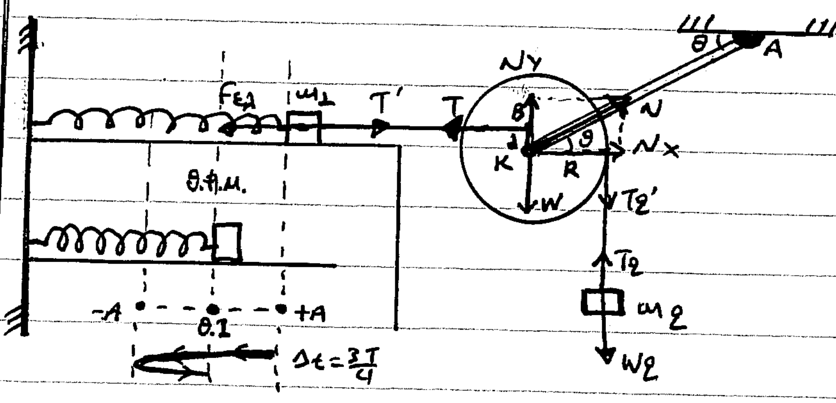
$I = \frac{1}{2} M R^2$

$m_1 = 4\,kg$, $k = 100\,N/m$

$(KB) = d = 0{,}1\,m$

$m_2 = 2\,kg$

Αβαρής ράβδος (ΚΑ)



A. Για να ισορροπεί στροφικά ($\Sigma\tau = 0$) η αβαρής ράβδος ΚΑ θα πρέπει οι δυνάμεις που της ασκούνται από την τροχαλία και το ταβάνι να έχουν την διεύθυνση της ράβδου. Επομένως η δύναμη N που ασκεί η αβαρής ράβδος ΚΑ στην τροχαλία έχει τη φορά του σχήματος.

• Σώμα m_2: $\Sigma F_y = 0 \Rightarrow T_2 - W_2 = 0 \Rightarrow T_2 = m_2 g \Rightarrow \underline{T_2 = 20N\ (=T_2')}$

• Τροχαλία: $\Sigma\tau_{(K)} = 0 \Rightarrow T\cdot d - T_2'\cdot R = 0 \Rightarrow T = T_2'\frac{R}{d} \Rightarrow \underline{T = 40N\ (=T')}$

$\Sigma F_x = 0 \Rightarrow N_x - T = 0 \Rightarrow \underline{N_x = 40N}$

$\Sigma F_y = 0 \Rightarrow N_y - W - T_2' = 0 \Rightarrow N_y = Mg + T_2' \Rightarrow \underline{N_y = 40N}$

Μέτρο: $N = \sqrt{N_x^2 + N_y^2} \Rightarrow N = \sqrt{2 N_x^2} \Rightarrow \boxed{N = 40\sqrt{2}\,N}$

Διεύθυνση: $\varepsilon\varphi\theta = \frac{N_y}{N_x} = 1$ ή $\boxed{\theta = 45^\circ}$

B. 1) α.α.τ. (m_1)

• $\Sigma F_x = 0 \Rightarrow T' - F_{ελ} = 0 \Rightarrow T' = k\cdot x_1 \Rightarrow \underline{x_1 = 0{,}4m\ (=A)}$

• $k = m_1\omega^2 \Rightarrow \omega = \sqrt{\frac{k}{m_1}} \Rightarrow \underline{\omega = 5\,rad/s}$

• $x = A\eta\mu(\omega t + \varphi_0) \xrightarrow[x=A]{t_0=0} A = A\eta\mu\varphi_0 \Rightarrow \eta\mu\varphi_0 = 1 = \eta\mu\frac{\pi}{2}$ ή $\underline{\varphi_0 = \frac{\pi}{2}\,rad}$

Άρα $\boxed{x = 0{,}4\eta\mu(5t + \frac{\pi}{2})}$, (S.I.)

2) $\left.\begin{array}{l} \Delta t = \frac{3T}{4} \\ T = 2\pi\sqrt{\frac{m_1}{k}} = 0{,}4\pi\,s \end{array}\right\} \Rightarrow \underline{\Delta t = 0{,}3\pi\,s}$

• Σώμα m_2: $\Sigma F_y = m_2 \cdot a_{cm} \Rightarrow w_2 - T_2 = m_2 a_{cm} \Rightarrow T_2 = m_2 g - m_2 a_{cm}$ (1)

• Τροχαλία: $\Sigma\tau = I a_{γων} \xrightarrow{a_{γων}=a_{cm}/R} T_2' \cdot R = \frac{1}{2}MR^2 \frac{a_{cm}}{R} \xrightarrow[(1)]{T_2'=T_2}$

$$\Rightarrow m_2 g - m_2 a_{cm} = \frac{M}{2} a_{cm} \Rightarrow a_{cm} = \frac{20}{3}\ m/s^2$$

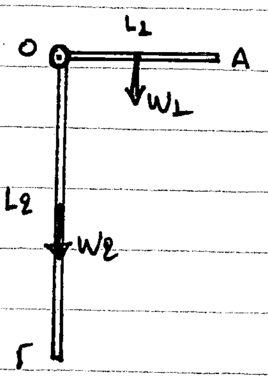
• $a_{γων} = \frac{a_{cm}}{R} \Rightarrow a_{γων} = \frac{100}{3}\ rad/s^2$

• $\frac{dK}{dt} = \Sigma F_y \cdot v_{cm} = m_2 \cdot a_{cm} v_{cm} \xrightarrow{v_{cm}=a_{cm}\Delta t = 2\pi\ m/s} \frac{dK}{dt} = 2\cdot\frac{20}{3}\cdot 2\pi\ J/s$

$$\Rightarrow \boxed{\frac{dK}{dt} = \frac{80\pi}{3}\ J/s}$$

• $L = I\omega \xrightarrow{\omega = a_{γων}\Delta t = 10\pi\ rad/s} L = \frac{1}{2} MR^2\omega \Rightarrow \boxed{L = 0{,}4\pi\ kgm^2/s}$

3.26 ΟΑ: $L_1 = 1m$, $m_1 = 2kg$
ΟΓ: $L_2 = 2m$, $m_2 = 1kg$



A. $I_O = I_O^{OA} + I_O^{OΓ}$ (1)

·Θεώρημα Steiner

$I_O^{OA} = I_{cm} + m_1 \frac{L_1^2}{4} \Rightarrow$

$I_O^{OA} = \frac{1}{12} m_1 L_1^2 + m_1 \frac{L_1^2}{4} \Rightarrow$

$I_O^{OA} = \frac{1}{3} m_1 L_1^2$

· Θεώρημα Steiner: $I_O^{OΓ} = I_{cm} + m_2 \frac{L_2^2}{4} = \frac{1}{12} m_2 L_2^2 + \frac{m_2 L_2^2}{4} \Rightarrow$

$$I_O^{OΓ} = \frac{1}{3} m_2 L_2^2$$

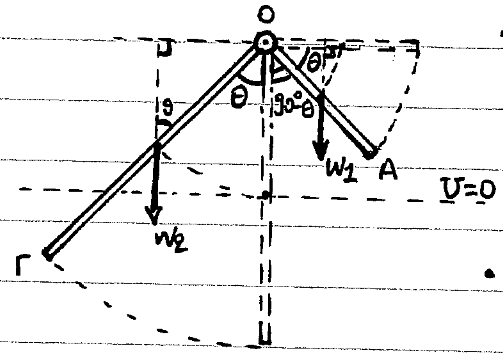
(1) $\Rightarrow I_O = \frac{1}{3} m_1 L_1^2 + \frac{1}{3} m_2 L_2^2 \Rightarrow \boxed{I_O = 2\ kg\cdot m^2}$

B.1)· $\Sigma\tau = I_O\cdot a_{γων} \Rightarrow w_1 \cdot \frac{L_1}{2} = I_O \cdot a_{γων} \Rightarrow a_{γων} = 5\ rad/s^2$

Άρα $\boxed{\frac{d\omega}{dt} = a_{γων} = 5\ rad/s^2}$

2) $\frac{dL}{dt} = \Sigma\tau = I_O\cdot a_{γων} \Rightarrow \boxed{\frac{dL}{dt} = 10\ kgm^2/s^2}$

Γ. Το σύστημα θα έχει μέγιστη ταχύτητα άρα και μέγιστη κινητική ενέργεια όταν $\Sigma\tau_{(O)}=0$. Έστω ότι αυτό συμβαίνει όταν έχει στραφεί κατά θ από την αρχική του θέση.



• $\Sigma\tau_{(O)}=0 \Rightarrow W_2 \frac{L_2}{2}\eta\mu\theta - W_1\frac{L_1}{2}\sigma\upsilon\nu\theta=0$

$\Rightarrow m_2 g \frac{L_2}{2}\eta\mu\theta = m_1 g\frac{L_1}{2}\sigma\upsilon\nu\theta \Rightarrow$

$\Rightarrow \varepsilon\varphi\theta = \frac{m_1L_1}{m_2L_2} \Rightarrow \varepsilon\varphi\theta = 1$ ή $\boxed{\theta=45^\circ}$

• Α.Δ.Μ.Ε. : $E_{MHX}^{\alpha\rho\chi} = E_{MHX}^{\tau\varepsilon\lambda} \Rightarrow K_{\alpha\rho\chi}+U_{\alpha\rho\chi} = K_{\tau\varepsilon\lambda}+U_{\tau\varepsilon\lambda} \Rightarrow$

$\Rightarrow m_1 g\frac{L_2}{2} = K + m_1 g\left(\frac{L_2}{2}-\frac{L_1}{2}\eta\mu\theta\right) + m_2 g\left(\frac{L_2}{2}-\frac{L_2}{2}\sigma\upsilon\nu\theta\right) \Rightarrow$

$\xrightarrow{\theta=45^\circ} K_{max} = \left[20-20\left(1-\frac{\sqrt2}{4}\right)-10\left(1-\frac{\sqrt2}{2}\right)\right] J \Rightarrow$

$\Rightarrow K_{max} = (20-20+5\sqrt2-10+5\sqrt2) J \Rightarrow K_{max} = (10\sqrt2-10) J \Rightarrow$

$\xrightarrow{\sqrt2\simeq1,4} \boxed{K_{max}=4 J}$

Δ. Α.Δ.Μ.Ε.: $E_{MHX}^{\alpha\rho\chi} = E_{MHX}^{\tau\varepsilon\lambda} \Rightarrow K_{\alpha\rho\chi}+U_{\alpha\rho\chi} = K_{\tau\varepsilon\lambda}+U_{\tau\varepsilon\lambda} \Rightarrow$

$\Rightarrow m_1 g\frac{L_2}{2} = m_1 g\left(\frac{L_2}{2}-\frac{L_1}{2}\eta\mu\theta\right) + m_2 g\left(\frac{L_2}{2}-\frac{L_2}{2}\sigma\upsilon\nu\theta\right) \Rightarrow$

$\Rightarrow 20 = 20\left(1-\frac{1}{2}\eta\mu\theta\right) + 10(1-\sigma\upsilon\nu\theta) \Rightarrow$

$\Rightarrow 20 = 20 - 10\eta\mu\theta + 10 - 10\sigma\upsilon\nu\theta \Rightarrow 10(\eta\mu\theta+\sigma\upsilon\nu\theta) = 10 \Rightarrow$

$\Rightarrow \eta\mu\theta+\sigma\upsilon\nu\theta = 1 \Rightarrow \eta\mu\theta = 1-\sigma\upsilon\nu\theta$ (1)

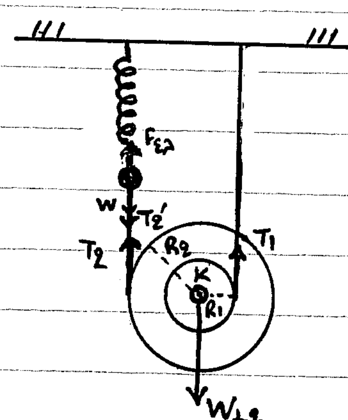
• Ισχύει $\eta\mu^2\theta+\sigma\upsilon\nu^2\theta = 1 \xrightarrow{(1)} (1-\sigma\upsilon\nu\theta)^2+\sigma\upsilon\nu^2\theta = 1 \Rightarrow$

$\Rightarrow 1-2\sigma\upsilon\nu\theta+\sigma\upsilon\nu^2\theta+\sigma\upsilon\nu^2\theta = 1 \Rightarrow 2\sigma\upsilon\nu^2\theta - 2\sigma\upsilon\nu\theta = 0 \Rightarrow$

$\Rightarrow 2\sigma\upsilon\nu\theta(\sigma\upsilon\nu\theta-1)=0 \Rightarrow \begin{cases} \sigma\upsilon\nu\theta=0 & \text{ή } \boxed{\theta=90^\circ} \\ \sigma\upsilon\nu\theta=1 & \text{ή } \theta=0^\circ \end{cases}$

3.27

$m_1 = 1 kg$, $R_1 = 0,1 m$
$m_2 = 2 kg$, $R_2 = 0,2 m$
$m = 1 kg$, $k = 100 N/m$



Α. Το σύστημα των δύο δίσκων ισορροπεί:

• $\Sigma\tau_{(K)}=0 \Rightarrow T_1\cdot R_1 - T_2\cdot R_2 = 0$

$\Rightarrow T_1 = 2T_2$ (1)

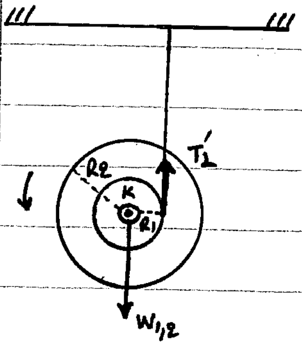
• $\Sigma F_y = 0 \Rightarrow T_1+T_2-W_{1,2}=0 \Rightarrow$

$\xrightarrow{(1)} 3T_2 = (m_1+m_2)g \Rightarrow \boxed{T_2=10N}$

$(1) \Rightarrow \boxed{T_1=20N}$

Β. Η ροπή αδράνειας του συστήματος των δύο δίσκων ως προς το Κ είναι:

$I_K = \frac{1}{2}m_1R_1^2+\frac{1}{2}m_2R_2^2 \Rightarrow I_K = (0,005+0,04) kg m^2 \Rightarrow \boxed{I_K = 0,045 kg m^2}$



1) • $\Sigma\tau = I_K \cdot a_{γων} \xrightarrow{a_{γων}=a_{cm}/R_1} T_1' \cdot R_1 = I_K \cdot \frac{a_{cm}}{R_1} \Rightarrow$

$\xRightarrow{S.I.} T_1' = 4,5\, a_{cm}$ (2)

• $\Sigma F = m_{1,2}\cdot a_{cm} \Rightarrow W_{1,2} - T_1' = m_{1,2}\, a_{cm} \xRightarrow{(2)}$

$\Rightarrow (M_1+M_2)g - 4,5\, a_{cm} = (M_1+M_2)\cdot a_{cm} \Rightarrow$

$\Rightarrow 30 - 4,5\, a_{cm} = 3\, a_{cm} \Rightarrow \boxed{a_{cm} = 4\, m/s^2}$

2) (2) $\Rightarrow T_1' = 18N$

• $Π\% = \frac{T_1 - T_1'}{T_1}\cdot 100\% \Rightarrow Π\% = \frac{20-18}{20}\cdot 100\% \Rightarrow$

$\Rightarrow \boxed{Π\% = 10\%}$

3) • $P_{T_1'} = \tau_{T_1'}\cdot\omega = T_1'\cdot R_1\cdot\omega \Rightarrow \omega = 80\,rad/s$

• $a_{γων} = \frac{a_{cm}}{R_1} \Rightarrow a_{γων} = 40\,rad/s^2$

• $\omega = a_{γων}\cdot t_1 \Rightarrow t_1 = 2s$

• $υ_{cm} = a_{cm}\, t_1 \Rightarrow υ_{cm} = 8\,m/s$

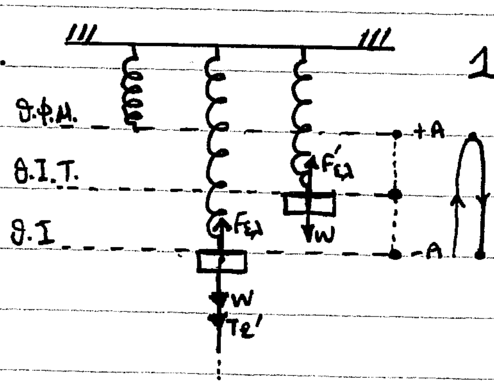
• $K = \frac{1}{2}(M_1+M_2)υ_{cm}^2 + \frac{1}{2} I_K\cdot\omega^2 \Rightarrow K = \frac{1}{2}\cdot 3\cdot 64\,J + \frac{1}{2}0,045\cdot 6400\,J$

$\Rightarrow \boxed{K = 240\,J}$

• $\theta = \frac{1}{2} a_{γων}\, t_1^2 \Rightarrow \theta = 80\,rad$

• $\theta = \frac{s}{R_1} \xrightarrow{s=\ell} \ell = \theta\cdot R_1 \Rightarrow \boxed{\ell = 8m}$

Γ.



1) • Θ.Ι.: $\Sigma F_y = 0 \Rightarrow F_{ελ} - W - T_2' = 0 \xRightarrow{T_2'=T_2}$

$\Rightarrow k\cdot y_1 = mg + T_2 \Rightarrow y_1 = 0,2m$

• Θ.Ι.Τ: $\Sigma F_y = 0 \Rightarrow F'_{ελ} - W = 0 \Rightarrow k\cdot y_2 = mg \Rightarrow$

$\Rightarrow y_2 = 0,1m$

• Άρα $A = y_1 - y_2 \Rightarrow A = 0,1m$

• $k = m\omega^2 \Rightarrow \omega = \sqrt{\frac{k}{m}} \Rightarrow \omega = 10\,rad/s$

• $y = Aημ(\omega t + φ_0) \xrightarrow[y=-A]{t_0=0} -A = Aημφ_0 \Rightarrow ημφ_0 = -1 = ημ\frac{3π}{2}$ ή $φ_0 = \frac{3π}{2}\,rad$

Άρα $\boxed{y = 0,1ημ(10t + \frac{3π}{2})}$, (S.I.)

2) Το σώμα m θα ακινητοποιηθεί στιγμιαία για δεύτερη φορά όταν φτάσει στη θέση $x=-A$ (σε χρόνο $Δt=T$).

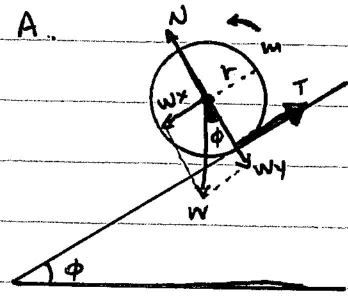
$\frac{|F_{ελ}|}{|F_{επ}|} = \frac{|k\cdot 2A|}{|k\cdot A|} \Rightarrow \boxed{\left|\frac{F_{ελ}}{F_{επ}}\right| = 2}$

3.28 $m=2kg$, $r=1m$, $\phi=30^\circ$, $x=2m$, $t=1s$

Δίσκος: $I_1=\frac{1}{2}MR^2$

Δακτύλιος: $I_2=MR^2$

A.



• $x=\frac{1}{2}\alpha_{cm}t^2 \Rightarrow \alpha_{cm}=4\frac{m}{s^2}$

• $\Sigma F_x=m\alpha_{cm} \Rightarrow w_x-T=m\alpha_{cm} \Rightarrow$

$\Rightarrow T=mg\eta\mu\phi-m\alpha_{cm} \Rightarrow T=2N$

• $\Sigma\tau=I\cdot\alpha_{\gamma\omega\nu} \xrightarrow{\alpha_{\gamma\omega\nu}=\alpha_{cm}/r=4\,rad/s^2}$

$T\cdot r=I\cdot\alpha_{\gamma\omega\nu} \Rightarrow \boxed{I=0{,}5\,kgm^2}$

B. • Δίσκος:

$\Sigma\tau=I_1\alpha_{\gamma\omega\nu} \xrightarrow{\alpha_{\gamma\omega\nu}=\alpha_{cm}/R} T\cdot R=\frac{1}{2}MR^2\frac{\alpha_{cm}}{R} \Rightarrow T=\frac{M}{2}\alpha_{cm}$ (1)

$\Sigma F=M\alpha_{cm} \Rightarrow w_x-T=M\alpha_{cm} \xrightarrow{(1)} Mg\eta\mu\phi-\frac{M}{2}\alpha_{cm}=M\alpha_{cm} \Rightarrow Mg\eta\mu\phi=\frac{3}{2}M\alpha_{cm}$

$\Rightarrow \alpha_{cm}^{(1)}=\frac{2}{3}g\eta\mu\phi$

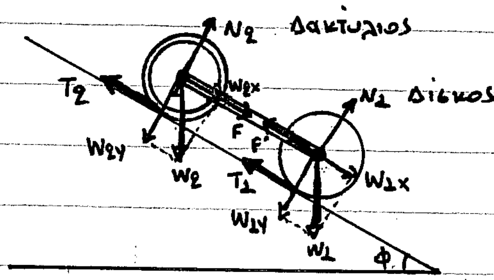
• Δακτύλιος:

$\Sigma\tau=I_2\alpha_{\gamma\omega\nu} \xrightarrow{\alpha_{\gamma\omega\nu}=\alpha_{cm}/R} TR=MR^2\frac{\alpha_{cm}}{R} \Rightarrow T=M\alpha_{cm}$ (2)

$\Sigma F=M\alpha_{cm} \Rightarrow w_x-T=M\alpha_{cm} \xrightarrow{(2)} Mg\eta\mu\phi=2M\alpha_{cm} \Rightarrow \alpha_{cm}^{(2)}=\frac{1}{2}g\eta\mu\phi$

Άρα $\boxed{\alpha_{cm}^{(1)}>\alpha_{cm}^{(2)}}$

Γ.



• $\frac{K_1}{K_2}=\frac{\frac{1}{2}MV_{cm}^2+\frac{1}{2}I_1\omega^2}{\frac{1}{2}MV_{cm}^2+\frac{1}{2}I_2\omega^2} \Rightarrow$

$\frac{K_1}{K_2}=\frac{\frac{1}{2}MV_{cm}^2+\frac{1}{2}\frac{1}{2}MR^2\omega^2}{\frac{1}{2}MV_{cm}^2+\frac{1}{2}MR^2\omega^2} \Rightarrow$

$\frac{K_1}{K_2}=\frac{\frac{3}{4}MV_{cm}^2}{MV_{cm}^2} \Rightarrow \boxed{\frac{K_1}{K_2}=\frac{3}{4}}$

Δ. • Δακτύλιος:

$\left.\begin{array}{l} \Sigma F_x=M\alpha_{cm} \Rightarrow F+W_{2x}-T_2=M\alpha_{cm} \Rightarrow F=T_2-Mg\eta\mu\phi+M\alpha_{cm} \\ \Sigma\tau=I_2\alpha_{\gamma\omega\nu} \Rightarrow T_2\cdot R=MR^2\frac{\alpha_{cm}}{R} \xrightarrow{S.I.} T_2=1{,}4\alpha_{cm} \end{array}\right\} \Rightarrow$

$\Rightarrow F=2{,}8\alpha_{cm}-7$ (3)

• Δίσκος

$\left.\begin{array}{l} \Sigma F_x=M\alpha_{cm} \Rightarrow W_{1x}-F'-T_1=M\alpha_{cm} \Rightarrow F'=Mg\eta\mu\phi-M\alpha_{cm}-T_1 \\ \Sigma\tau=I_1\alpha_{\gamma\omega\nu} \Rightarrow T_1R=\frac{1}{2}MR^2\frac{\alpha_{cm}}{R} \Rightarrow T_1=0{,}7\alpha_{cm} \end{array}\right\} \Rightarrow$

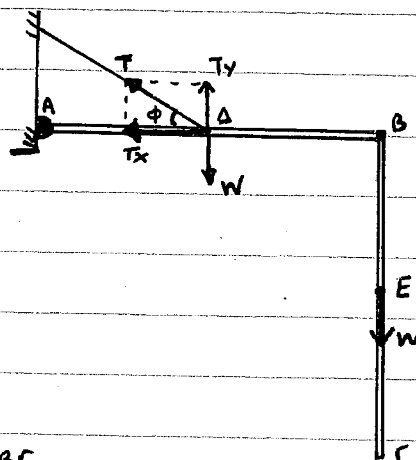
$\Rightarrow F'=7-2{,}1\alpha_{cm}$ (4)

• $F = F' \xrightarrow[(4)]{(3)} 2{,}8\alpha_{cm} - 7 = 7 - 2{,}1\alpha_{cm} \Rightarrow 4{,}9\alpha_{cm} = 14 \Rightarrow$

$\Rightarrow \alpha_{cm} = \frac{14}{4{,}9}\ m/s^2$

• $(3) \Rightarrow F = \left(2{,}8 \cdot \frac{14}{4{,}9} - 7\right) N \Rightarrow \boxed{F = 1N}$

3.29 $L = 0{,}6m$, $m = 1kg$, $\phi = 30^\circ$, Δ: μέσον



A. Το σύστημα ισορροπεί

$\Sigma\tau_{(A)} = 0 \Rightarrow T_y \cdot \frac{L}{2} - w\frac{L}{2} - wL =$

$\Rightarrow T \cdot \eta\mu\phi \cdot \frac{L}{2} = mg\frac{L}{2} + mgL \Rightarrow$

$\Rightarrow \boxed{T = 60N}$

B. • $I_A = I_A^{AB} + I_A^{B\Gamma}$ (1)

• θεώρημα Steiner: $I_A^{AB} = I_{cm} + m\left(\frac{L}{2}\right)^2 = \frac{1}{12}mL^2 + m\frac{L^2}{4} = \frac{1}{3}mL^2$

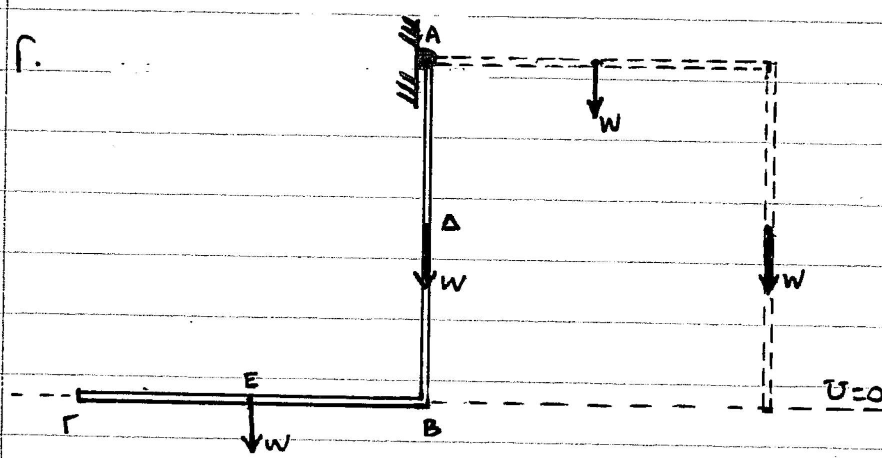
• θεώρημα Steiner: $I_A^{B\Gamma} = I_{cm} + m(AE)^2$

πυθαγ. θεωρ.: $(AE)^2 = (AB)^2 + (BE)^2 = L^2 + \frac{L^2}{4} = \frac{5L^2}{4}$ $\Rightarrow$

$\Rightarrow I_A^{B\Gamma} = \frac{1}{12}mL^2 + \frac{5L^2}{4}m = \frac{16mL^2}{12} = \frac{4mL^2}{3}$

$(1) \Rightarrow I_A = \frac{1}{3}mL^2 + \frac{4}{3}mL^2 = \frac{5}{3}mL^2 \Rightarrow \boxed{I_A = 0{,}6\ kg \cdot m^2}$

Γ.



• $\Sigma\tau = I_A \cdot \alpha_{\gamma\omega\nu} \Rightarrow$

$w \cdot \frac{L}{2} + wL = I_A \cdot \alpha_{\gamma\omega\nu} \Rightarrow$

$mg\frac{L}{2} + mgL = I_A \cdot \alpha_{\gamma\omega\nu} \Rightarrow$

$\alpha_{\gamma\omega\nu} = 15\ rad/s^2$

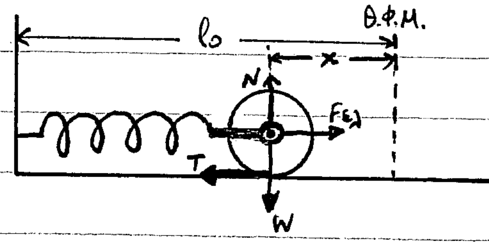
Άρα $\boxed{\frac{d\omega}{dt} = 15\ rad/s^2}$

Δ. • $\frac{dL}{dt} = \Sigma\tau = -w \cdot \frac{L}{2}$ ή $\boxed{\frac{dL}{dt} = -3\ kg\,m^2/s^2}$

• Α.Δ.Μ.Ε.: $E_{MHX}^{\alpha\rho\chi} = E_{MHX}^{\tau\varepsilon\lambda} \Rightarrow K_{\alpha\rho\chi} + U_{\alpha\rho\chi} = K_{\tau\varepsilon\lambda} + U_{\tau\varepsilon\lambda} \Rightarrow$

$mgL + mg\frac{L}{2} = K + mg\frac{L}{2} \Rightarrow \boxed{K = 6J}$

3.30 $m = 2kg$, $R = 0,1m$, $k = 300\,N/m$, $x = 0,2m$



A. • $\Sigma\tau = I\cdot\alpha_{γων} \xrightarrow{\alpha_{γων}=\alpha_{cm}/R} T\cdot R = \frac{1}{2} m R^2 \frac{\alpha_{cm}}{R} \Rightarrow \alpha_{cm} = \frac{2T}{m}$ (1)

• $\Sigma F = m\alpha_{cm} \Rightarrow F_{ελ} - T = m\alpha_{cm} \xrightarrow{(1)} F_{ελ} - T = 2T \Rightarrow T = \frac{F_{ελ}}{3}$ (2)

• $\Sigma F = T - F_{ελ} \xrightarrow{(2)} \Sigma F = \frac{F_{ελ}}{3} - F_{ελ} \Rightarrow \Sigma F = -\frac{2}{3}k\cdot x$

Άρα το κέντρο μάζας του κυλίνδρου θα εκτελέσει α.α.τ. με $D = \frac{2}{3}k = 200\,N/m$.

• $T = 2\pi\sqrt{\frac{m}{D}} \Rightarrow \boxed{T = 0,2\pi\, s}$

B. • $E = K + U \Rightarrow K = E - U \Rightarrow K = \frac{1}{2}DA^2 - \frac{1}{2}Dx^2 \xrightarrow{x=-A/2} K = \frac{1}{2}DA^2 - \frac{1}{2}D\frac{A^2}{4}$
$\Rightarrow K = E - \frac{E}{4} \Rightarrow K = \frac{3E}{4}$ (1)

• $\frac{K_{μετ}}{K_{στρ}} = \frac{\frac{1}{2}m v_{cm}^2}{\frac{1}{2}I\omega^2} = \frac{\frac{1}{2}m v_{cm}^2}{\frac{1}{2}\frac{1}{2}mR^2\omega^2} = 2$ ή $K_{μετ} = 2\cdot K_{στρ}$ (2)

• $K = K_{μετ} + K_{στρ} \xrightarrow{(2)} K = 3K_{στρ} \xrightarrow{(1)} \frac{3E}{4} = 3K_{στρ} \Rightarrow K_{στρ} = \frac{E}{4}$

Άρα $\Pi\% = \frac{K_{στρ}}{E}\cdot 100\% \Rightarrow \Pi\% = \frac{\frac{E}{4}}{E}\cdot 100\% \Rightarrow \boxed{\Pi\% = 25\%}$

Γ. • $t_0 = 0$, $x_0 = 0,2m$, $v_0 = 0$: $A = x_0 = 0,2m$

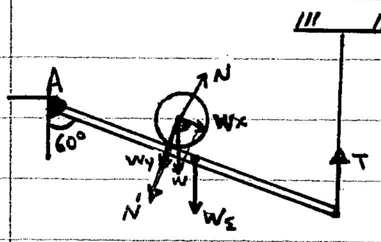
• $D = m\omega^2 \Rightarrow \omega = \sqrt{\frac{D}{m}} \Rightarrow \omega = 10\, rad/s$, $\omega = \frac{2\pi}{T} \Rightarrow T = 0,2\pi\, s$

• $x = A\eta\mu(\omega t + \phi_0) \xrightarrow[x=-A]{t_0=0} -A = A\eta\mu\phi_0 \Rightarrow \eta\mu\phi_0 = -1 = \eta\mu\frac{3\pi}{2}$ ή $\phi_0 = \frac{3\pi}{2} rad$

• Α.Δ.Ενέργειας ταλάντωσης για το κέντρο μάζας του κυλίνδρου:
$E = K_{max} \Rightarrow \frac{1}{2}DA^2 = \frac{1}{2}m v_{max}^2 \Rightarrow v_{max} = A\sqrt{\frac{D}{m}} \Rightarrow v_{max} = 2\, m/s$

Άρα $\boxed{v = 2\sigma\upsilon\nu\left(10t + \frac{3\pi}{2}\right)}$ (3) (S.I.)

Δ. • (3) $\xrightarrow{t = \frac{5T}{12} = \frac{\pi}{12}s} v = 2\sigma\upsilon\nu\left(\frac{10\pi}{12} + \frac{3\pi}{2}\right) \Rightarrow v = 2\sigma\upsilon\nu\left(\frac{5\pi}{6} + \frac{9\pi}{6}\right) \Rightarrow$
$\Rightarrow v = 2\sigma\upsilon\nu\frac{14\pi}{6} \Rightarrow v = 2\sigma\upsilon\nu\left(2\pi + \frac{\pi}{3}\right) \Rightarrow v = 2\sigma\upsilon\nu\frac{\pi}{3} \Rightarrow v_{cm} = 1\, m/s$,
οπότε $v_{cm} = \omega R \Rightarrow \omega = 10\, rad/s$.



• Στον κύλινδρο ασκούνται οι δυνάμεις:

- $W = mg = 20N$ με $W_x = W\sigma\upsilon\nu 60^\circ = 10N$ και $W_y = W\eta\mu 60^\circ = 10\sqrt{3}\,N$
- $\Sigma F_y = 0 \Rightarrow N = W_y = 10\sqrt{3}N$
- $\Sigma F_x = m\alpha_{cm} \Rightarrow W_x = m\alpha_{cm} \Rightarrow \underline{\alpha_{cm} = 5m/s^2}$
- $\Sigma\tau = 0$, ομαλή κυκλική κίνηση ($\omega = 10\,rad/s$)

• Στη ράβδο ασκούνται οι δυνάμεις:

- $W_\Sigma = m_\Sigma \cdot g = 20N$, $N' = 10\sqrt{3}N\ (=N)$ αντίδραση της δύναμης N, T.
- $\Sigma\tau_{(A)} = 0 \Rightarrow T \cdot l\eta\mu 60^\circ + W_\Sigma \cdot x\,\eta\mu 60^\circ - N'\frac{l}{2} = 0 \Rightarrow$

$\xrightarrow{T=T_{\theta\rho}} 25 \cdot 2 \cdot \frac{\sqrt{3}}{2} + 20 \cdot x \cdot \frac{\sqrt{3}}{2} - 10\sqrt{3} \cdot \frac{2}{2} = 0 \Rightarrow$

$\Rightarrow 10x = 15 \Rightarrow \boxed{x = 1{,}5m}$

$x = \upsilon_0 t + \frac{1}{2}\alpha_{cm} t^2 \Rightarrow 1{,}5 = t + 2{,}5t^2 \Rightarrow \underline{2{,}5t^2 + t - 1{,}5 = 0}$, $\Delta = 16$, $\begin{cases} \boxed{t_1 = 0{,}6\,s} \text{ δεκ} \\ t_2 = -1s \text{ απ} \end{cases}$

Ε. • $0 - \frac{\pi}{12}s$: $\upsilon = 2\sigma\upsilon\nu\left(10t + \frac{3\pi}{2}\right)$, (S.I.)

• $\left(\frac{\pi}{12} - \frac{\pi}{12} + 0{,}6\right)s$: $\upsilon = \upsilon_0 + \alpha_{cm}\Delta t \Rightarrow \upsilon = 1 + 5\left(t - \frac{\pi}{12}\right)$, (S.I.)